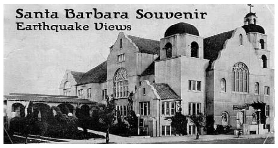

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MAXIMUM LIKELIHOOD PARAMETER ESTIMATION FOR SURFACE WAVES: APPLICATION TO AMBIENT VIBRATIONS

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ABSTRACT

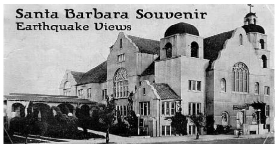
The analysis of ambient vibrations represents a valuable tool in seismic microzonation, engineering seismology, and other fields. An extensively used approach for the study of ambient vibrations is the use of array processing techniques. We have developed a novel technique for the analysis of the seismic wave field and show an application to the analysis of ambient vibrations. We derived maximum likelihood estimators for the parameters of the different wave types, considering all the measurements simultaneously. Our method allows us to separate the contribution of Love and Rayleigh waves as well as fundamental and higher modes. We assess the performance on SESAME synthetic models. We show that the proposed approach allows to detect weaker signals from higher modes, even when they are not visible with traditional techniques. This leads to a more accurate estimation of the dispersion curves, potentially over a broader frequency range and including larger portion of higher modes. In addition, we estimate Rayleigh wave ellipticity with a maximum likelihood estimator and estimate the retrograde vs. prograde behavior of the particle motion.

INTRODUCTION

The analysis of ambient vibrations represents a valuable tool in seismic microzonation, engineering seismology, and other fields. An extensively used approach for the study of ambient vibrations is the use of array processing techniques (Fäh et al. 2008, Cornou et al. 2003). Array processing techniques currently in use present several limitations, such as: measurements from different components of the seismometer are processed separately; wave field parameters are not estimated jointly; superposition of different wave phenomena is not accounted for.

We have developed a novel technique for the analysis of the seismic wave field and show an application to the analysis of ambient vibrations (Maranò et al. 2011b, submitted). The proposed technique relies on a particular type of probabilistic graphical model called factor graph. We derived maximum likelihood estimators for the parameters of the different wave types, considering all the measurements simultaneously. Our method works in the time domain and addresses wave superposition. This enables us to separate the contribution of Love and Rayleigh waves as well as fundamental and higher modes.

We assess the performance of the described technique on the SESAME synthetic dataset (Bonnefoy-Claudet et al. 2006). We show that the proposed approach allows to detect weaker signals from higher modes, even when they are not visible with traditional techniques. This leads to a more accurate estimation of the dispersion curves, potentially over a broader frequency range and including larger portion of higher modes. In addition, we estimate Rayleigh wave ellipticity with a maximum likelihood estimator and estimate the retrograde vs. prograde behavior of the particle motion.



OVERVIEW OF THE METHOD

The proposed technique performs maximum likelihood (ML) estimation of wave field parameters. The technique allows to estimate wave parameters (i.e., amplitude, phase, wavenumber, direction of arrival, and polarization) of Love and Rayleigh waves. We consider a monochromatic wave, and model all the measurement and all the parameters jointly. In contrast with other array processing techniques used in seismology *(*e.g. Fäh et al. 2008*)*, our method uses all the measurements recorded at three-components sensors jointly, thus enabling for better performance. Notably, our modeling enables the retrieval of the sense of rotation of the Rayleigh wave particle.

In addition, it is possible to model the simultaneous presence of multiple waves. The algorithm initially models a single wave and subsequently increase the number of waves modeled gradually. Each wave modeled can be either a Rayleigh or a Love waves. The contribution of each wave to the wave field are separated and it is possible to improve the parameter estimation.

The technique relies on factor graphs, a type of probabilistic graphical models (Loeliger et al. 2007). We use a factor graph to model the probability density function of the measurement and the wave parameters. Using the sum-product algorithm it is possible to compute the likelihood of the observations. A detailed description of the ML method can be found in (Maranò et al. 2011a).

NUMERICAL RESULTS

We assess the performance of the described technique on the SESAME structural model M10.2. The model is a two layers over an half-space. Details of the structural model are described in Table 1. Being the structural model known it is possible to compute numerically the theoretical dispersion curves for both Rayleigh and Love waves. In the following pictures the fundamental mode is shown with the solid red line, the first higher mode in dashed blue line, and the second higher mode by the dashed-dotted magenta line. The same convention of colors applies for Rayleigh ellipticity curves.
To produce the numerical results, we use an array of 14 three-component sensors, which geometry is shown in Fig. 1. The total recording, having duration of 400 seconds is split in non-overlapping windows of 2.5 seconds. The wavenumber estimated in the different windows is combined by means of the Parzen window method and is shown in the pictures as a gray scale, darker in presence of more estimates, lighter when less estimates are found.

Table 1. Details of the SESAME model M10.2

	V_P	V_S	Q_P	Q_S	Density	Thickness
	[m/s]	[m/s]			[kg/m^3]	[m]
Layer 1	1350	250	50	25	1900	18
Layer 2	1350	333	50	25	1900	18
Layer 3	2000	1000	100	50	2500	Infinite

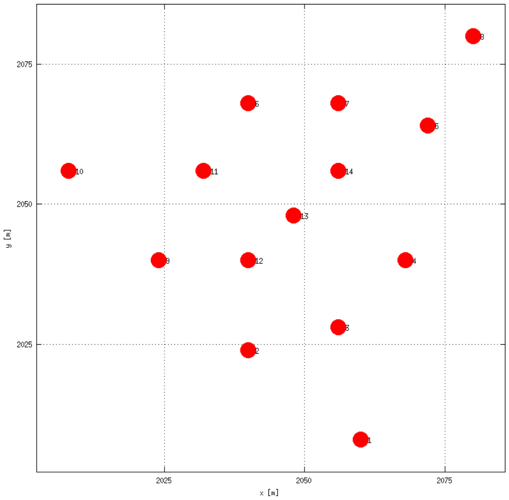


Fig. 1. Geometry of the 14 sensor array used in the processing.

Figure 2 shows the dispersion relations as found using the three-components beamforming proposed in *Fäh et al. (2008)*. The dispersion relation is shown in wavenumber vs. frequency, a representation equivalent to the more common velocity vs. frequency. The first two subfigures refer to Rayleigh waves, and show the estimated wavenumbers obtained from the processing of the radial and vertical components. The third subfigure refers to Love waves and show the dispersion curve obtained from the processing of the transverse component.

In Fig. 3 results from the proposed technique are presented. The two upper figures show the dispersion curve for Love (left) and Rayleigh (right) obtained by modeling a single wave. For each time window the algorithm chooses adaptively whether to model a Love or a Rayleigh wave based on the energy of the wave. It is possible to see how the variance is reduced. The two lower subfigures of Fig. 3 refer to the joint modeling of three waves. It is possible to notice how the Rayleigh wave higher modes are more clearly visible.

In Fig. 4 we show the result for the estimation of the Rayleigh wave ellipticity obtained with the ML technique. The two leftmost figures show the ellipticity in the usual H/V representation. The H/V is the ratio of the amplitude of the Rayleigh wave on the horizontal and vertical component. The two rightmost pictures show the ellipticity of the Rayleigh wave as the ellipticity angle xi, which is related to H/V as

$$H/V = \tan(xi) \tag{1}$$

The advantage of such representation is that it allows to distinguish between retrograde and prograde particle motion. Values of xi between 0 and pi/2 correspond to prograde particle motion, values between -pi/2 and 0 to retrograde particle motion.

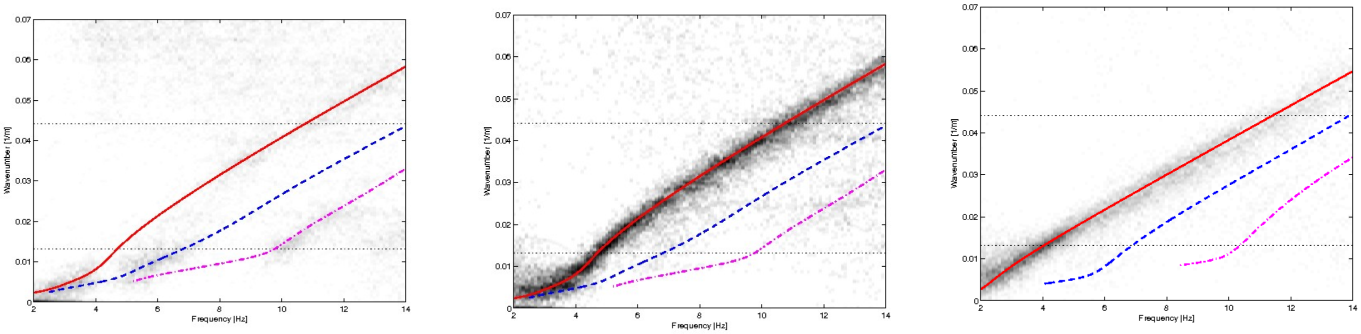


Fig. 2. Dispersion relation as retrieved using three components beamforming (Fäh et al. 2008). From left to right: radial component, vertical component, and transverse component.

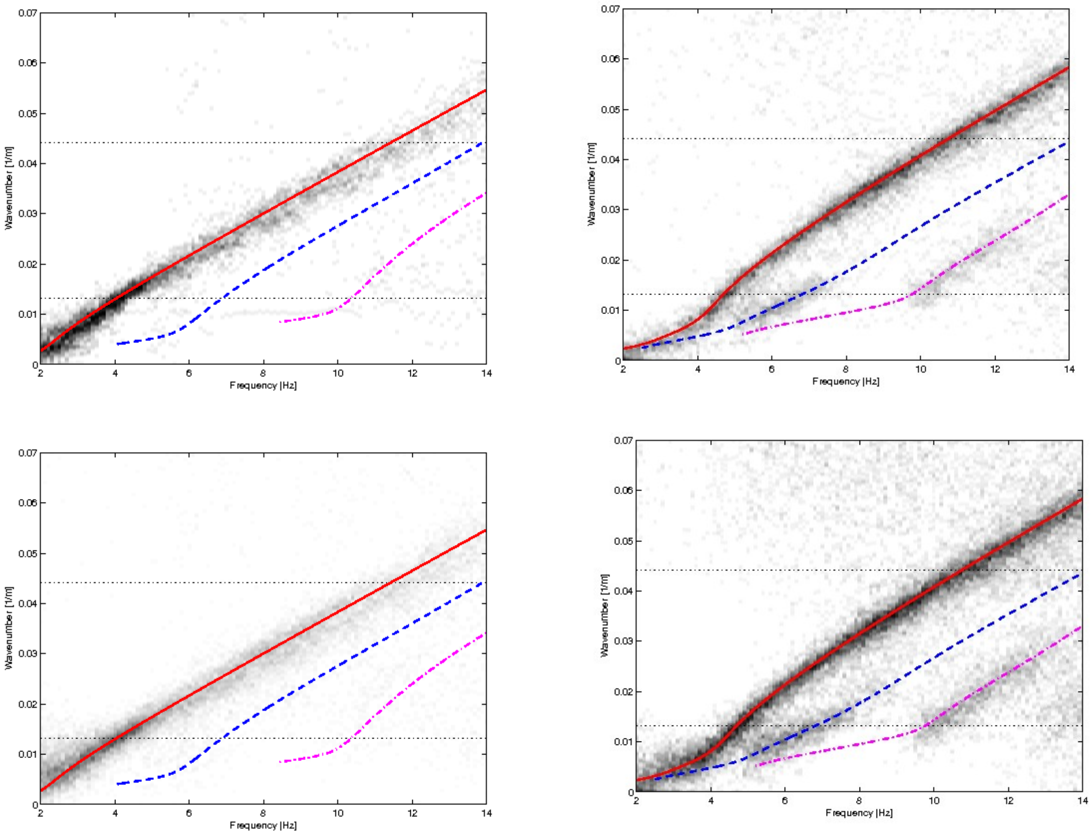


Fig. 3. Joint modeling of multiple waves. Top row show the modeling of one wave for each time window. Bottom row the modeling of three waves. On the left Love wave wavenumber is shown, on the right Rayleigh wave.

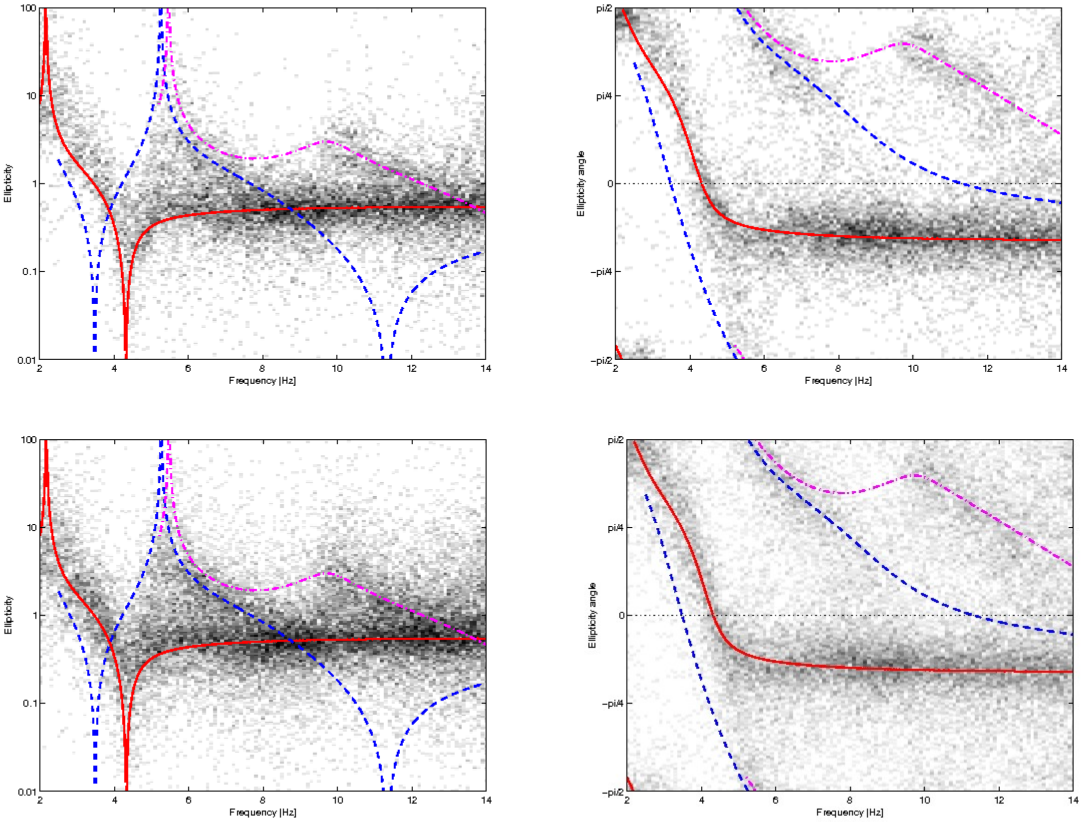


Fig. 4. Rayleigh wave ellipticity. Top pictures, refers to single wave modeling, bottom to the joint modeling of three waves. On the left Rayleigh wave ellipticity is represented trough the usual H/V representation. On the right, with the ellipticity angle xi.

CONCLUSIONS

We considered a ML estimator for wave field parameters of surface waves. The technique models jointly all the measurements from three-components sensors and all the wave field parameters. The technique allows to model jointly the presence of multiple waves. We show an application of the method to ambient vibrations. We use a SESAME structural model and show how it is possible to retrieve Rayleigh and Love dispersion curves. We show the retrieval of Rayleigh wave ellipticity with information about the prograde/retrograde particle motion and the modeling of multiple waves.

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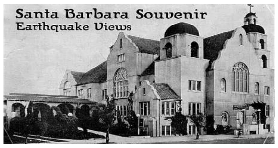

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