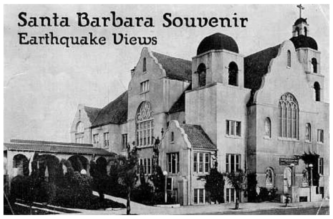

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THE EFFECT OF LATERAL HETEROGENEITY ON H/V SPECTRAL RATIO OF MICROTREMORS CONFIRMED FROM OBSERVATION AND SYNTHETICS

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ABSTRACT

Horizontal-to-Vertical spectral ratios of microtremors (HVRM) have been traditionally interpreted as representing either directly the S wave amplification or the Rayleigh wave ellipticity for a horizontally layered structure. However, based on the diffuse field theory we propose as an alternative assumption that the microtremors belong to a diffuse field. Under this condition the HVRM corresponds to the square root of the ratio of the imaginary part of horizontal displacement for a horizontally applied unit harmonic load on the free surface and the imaginary part of vertical displacement for a vertically applied unit harmonic load. For a laterally heterogeneous underground structure a numerical approach is needed to interpret HVRM. As observational evidence of non 1D HVRM, we discover significant directional dependency at a site in Uji campus, Kyoto University, Japan, where the bedrock depth varies from east to west from 250m to 420m within 1 km. The observed microtremor NS/UD spectral ratios are quite stable and have only one peak at around 0.5 Hz. On the other hand, the EW/UD spectral ratios are smaller in amplitude and have higher peak frequencies and sometimes two peaks. We perform analyses using point source in both the 1D and 2D models.

INTRODUCTION

Horizontal-to-Vertical (H/V) spectral ratios of microtremors (HVRM) are useful to establish the dominant frequency of a site and therefore are useful for site characterization and micorzonation. They have been traditionally interpreted as representing either the Rayleigh wave ellipticity (Lermo and Chávez-García 1994; Malischewsky and Scherbaum 2004) or the S wave amplification directly (see e.g. Nakamura 2000; Bonnefoy-Claudet *et al*. 2008; Herat 2008) for a horizontal stack of layers. However, based on the diffuse field theory (Perton *et al.*, 2009), the HVRM corresponds to the square root of the ratio of the imaginary part of horizontal displacement for a horizontally applied unit harmonic load, $\mathrm{Im}[G_{11}]$, and the imaginary part of vertical displacement for a vertically applied unit harmonic load, $\mathrm{Im}[G_{33}]$ (Sánchez-Sesma *et al*., 2011).

In this paper, we explore the applicability of the diffuse field concepts to analyze microtremor records when there is lateral heterogeneity. For a sufficiently flat, horizontally layered structure, we can easily calculate the theoretical Green's function for that 1D model. Therefore, if we observe microtremors we can invert the underground structure below that site by using theoretical point source solution. On the other hand, for a laterally heterogeneous underground structure, the horizontal reflection responses are different (Uebayashi, 2003) and in order to interpret microtremor H/V spectral ratios for sites with lateral heterogeneity, a numerical approach is needed. We can use methods like 3D finite-difference method (FDM) to study cases of laterally heterogeneous elastic layers over a half-space.

We observed directional dependency of microtremor H/V ratios and performed several analyses by using the proposed idea of this study. The directional dependence can be considered to be the result of non 1D subsurface geology.

After preliminary 1D analysis using a model with a horizontal stack of layers, we will perform 3D point source analysis to see directional dependence in the Green's function similar to the observation using FDM.

MICROTREMOR OBSERVATION

We performed continuous microtremor measurements to obtain data in order to understand the velocity structure of Uji campus of Kyoto University, Japan, through analysis of H/V spectral ratios.

Conditions of Measurement and Analysis

We used portable accelerometer SMAR-6A3P with external battery as shown in Fig. 1 (a) for each site. Four sites were chosen for our deployment within the Uji campus (see Fig. 1b). In fact, the SMAR-6A3P consists of three accelerometers for two horizontal and one vertical component combined with a LS-8000WD data logger. We made observations with the following conditions; 100 Hz sampling, 500 times amplified, time correction with GPS, and consecutive time duration of 15 or 30 minutes with interval of the same duration. From the observed microtremor time series, we took out 40.96 second time window sections overlapping half of the time window, resulting of 42 time sections from 15 minutes and 84 time sections from 30 minutes of observation for the analysis. Then we calculated averaged Horizontal-to-Vertical spectral ratios for each time section for NS and EW components separately and not by averaging the two horizontal components as done in conventional H/V spectral ratio studies. Finally, we averaged the H/V spectral ratio for each day of measurement.

(a) Example of microtremor measurement site (site C)

(b) Map of observation sites

Fig. 1. Microtremor measurement in Uji campus of Kyoto University, Kyoto, Japan.
(a) Photo image of microtremor measurement site using portable accelerometer SMAR-6A3P and external battery (Silver box in the right side of photo is the SMAR-6A3P), (b) Map of observation sites in Uji campus ("X" shows the microtremor measurement sites).

Observed H/V Spectral Ratio

Figure 2 shows the average H/V spectral ratio for NS/UD and EW/UD components separately for data of June 21st, 2010, at sites A to D shown in Fig. 1b. From the figures, we can see that for every site the peak ratio of NS/UD component at about 4.0 is larger than that of EW/UD component at about 2.5 and the peak frequency of NS/UD component at 0.5Hz is smaller than that of EW/UD component at 0.6Hz. For data of June 23rd, 2010, shown in Fig. 3, the peak frequency for NS/UD component is same at 0.5Hz as Fig. 2, but for EW/UD component, a second peak at about 0.4Hz appears along with the 0.6Hz peak. The NS/UD component looks more stable compared to EW/UD component.

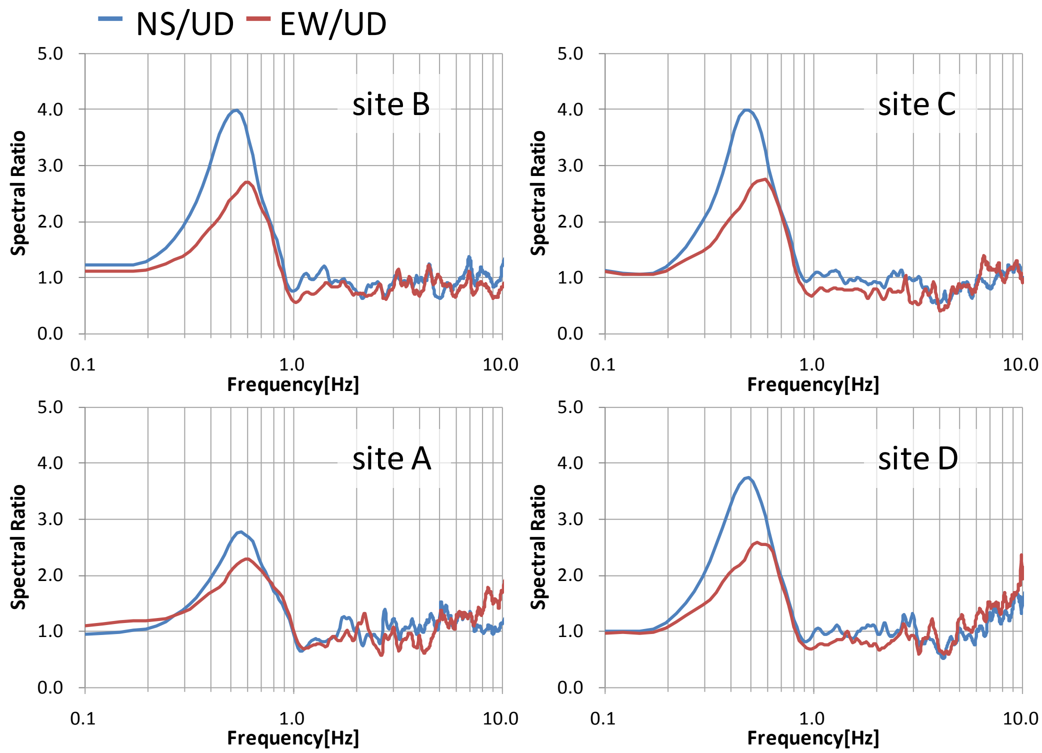


Fig. 2. Average H/V spectral ratio of June 21st, 2010, observed at sites A to D in Fig. 2.
For the peak ratio, NS/UD > EW/UD, and for the peak frequency, EW/UD > NS/UD.

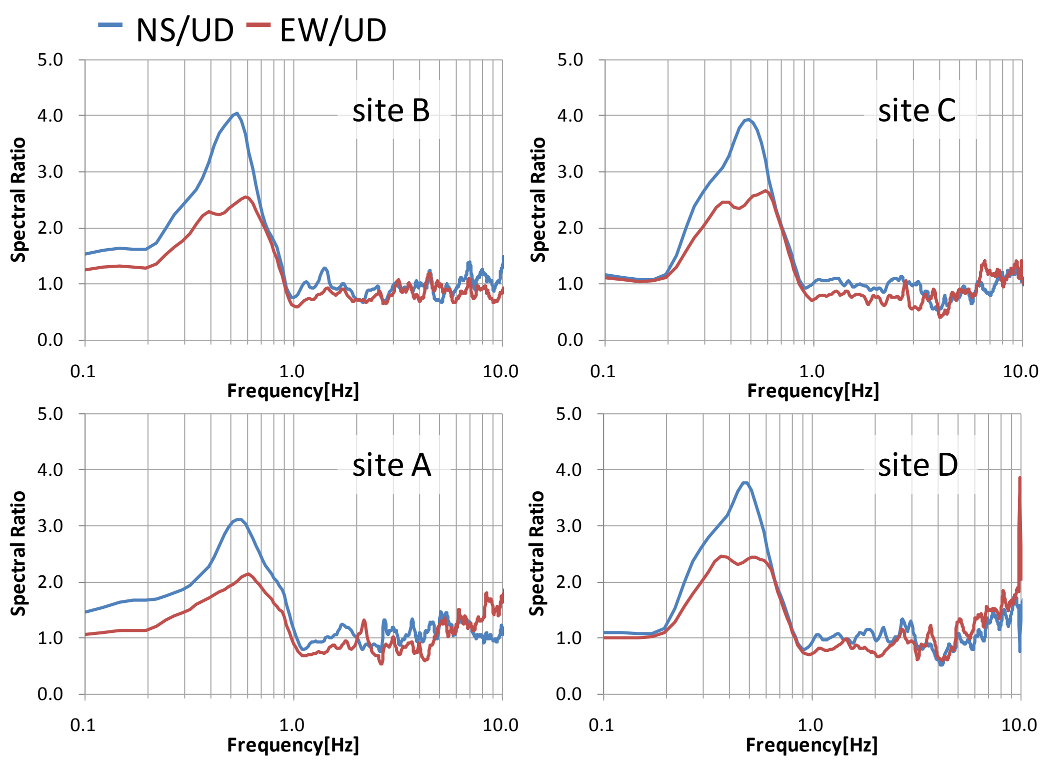


Fig. 3. Average H/V spectral ratio of June 23rd, 2010, observed at sites A to D in Fig. 2.
The overall characteristics are same as June 21st, but EW/UD has a second peak with lower frequency.

Previous survey and research by Koizumi *et al*. (2002) show that the bedrock is dipping east to west from the Obaku fault running north-to-south just east of the campus. The bedrock depth beneath Uji campus varies from 250m to 420m within 1km distance as shown in Fig. 4. In the figure, the peak frequency of H/V spectral ratios are listed for NS/UD and EW/UD components in red and blue, respectively. The alphabet corresponds to the name of the measurement sites in Fig. 1 (b). The peak frequency for the NS/UD component gets higher as the bedrock depth gets smaller, but for the EW/UD component it does not seem to depend on the depth of the bedrock. From this result, we may assume that the 2D basin structure is playing a role in the difference between the NS/UD and EW/UD components. In order to check the validity of the assumption, we checked the correspondence of the direction of the Obaku fault to the angle that the difference between two horizontal components becomes largest. By rotating the horizontal coordinate from 0 to 90 degrees we searched for the rotation angle that the ratio of amplitude of spectral ratio of NS/UD and EW/UD becomes smallest (Fig. 5a). As a result, the lowest ratio between NS/UD and EW/UD was around 5 to 15 degrees. In Fig. 5b, we can see that the fault trace of Obaku fault is about N13.2E, so that the two angles are in good agreement.

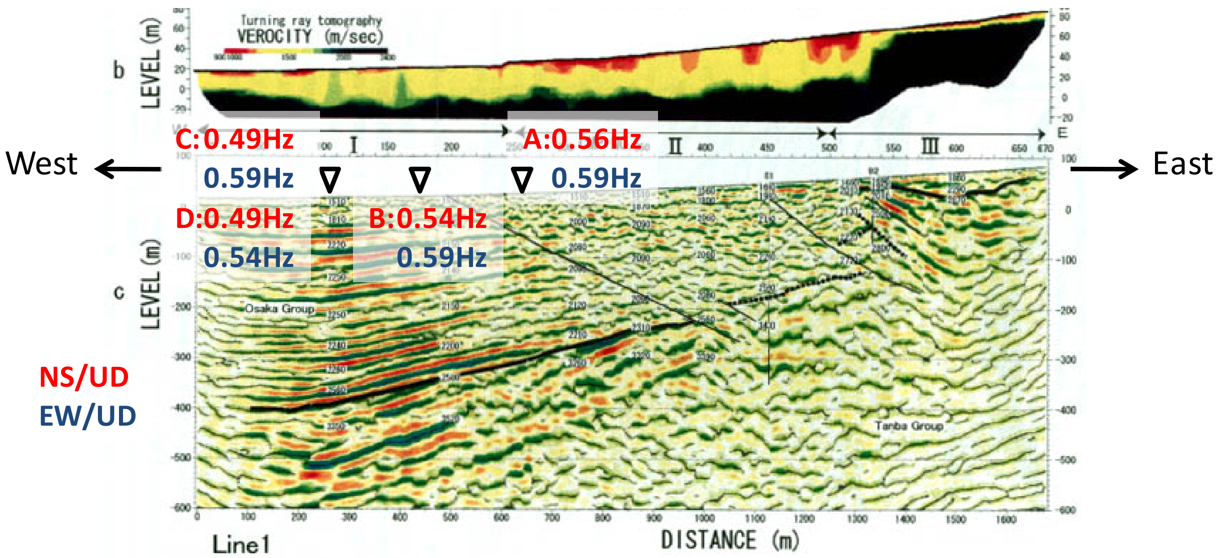
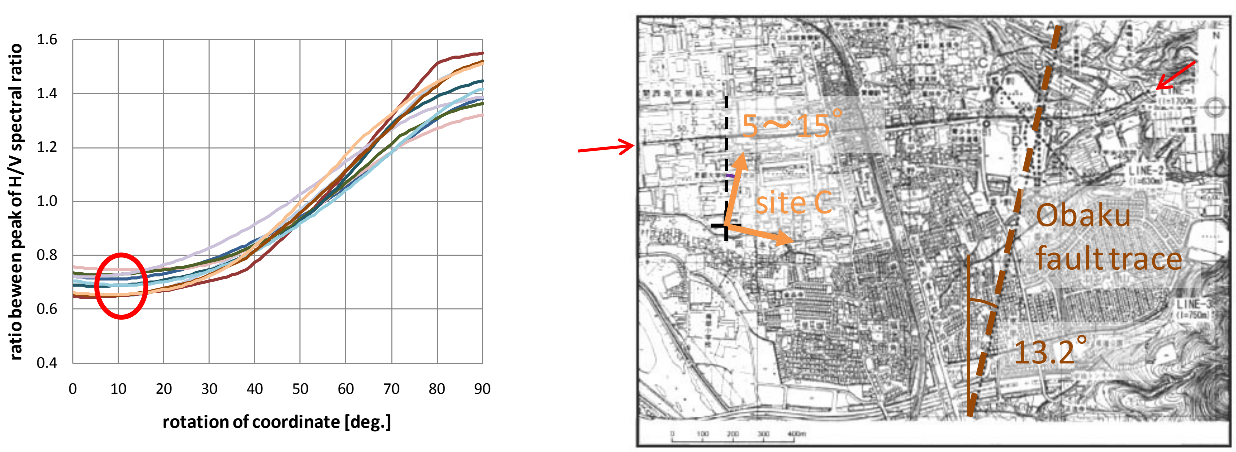


Fig. 4. Section profile beneath Uji campus along the survey line in Fig.6(b) (After Koizumi et al., 2002) and peak frequency of average H/V spectral ratio of June 21st, 2010, observed at sites A to D in Fig.2.



(a) Transition of difference of peak H/V spectral ratio according to rotation of coordinate

(b) Correspondence of rotation angle (after Koizumi *et al*., 2002)

Fig. 5. Correspondence of rotation angle for lowest difference of peak H/V spectral ratio to the Obaku fault just east of Uji campus. (a) Transition of peak H/V spectral ratio according to rotation of coordinate at site C, (b) Rotation angle for lowest ratio of peak H/V spectral ratios (5 – 15 deg.) show good correspondence to the strike of fault trace of Obaku fault (N13.2E). Red arrows in (b) denotes the location of survey line of Fig. 5.

THEORETICAL H/V SPECTRAL RATIO

In this section, we demonstrate our new proposed theory, that based on the diffuse field theory (Perton *et al.*, 2009) the microtremor H/V spectral ratio corresponds to the square root of the ratio of the imaginary part of horizontal displacement for a horizontally applied unit harmonic load, $\mathrm{Im}[G_{11}]$, and the imaginary part of vertical displacement for a vertically applied unit load, $\mathrm{Im}[G_{33}]$ (Sánchez-Sesma *et al.*, 2011). We will also show the comparison between theoretical and observed H/V spectral ratio.

Formulation of Theoretical H/V Spectral Ratio Based on Diffuse Field Theory

Within a 3D diffuse, equipartitioned field, the average cross correlations of displacement at points $\mathbf{x}_\mathrm{A}$ and $\mathbf{x}_\mathrm{B}$ can be written as:

$$\langle u_i(\mathbf{x}_\mathrm{A},\omega)u^*{}_j(\mathbf{x}_\mathrm{B},\omega)\rangle = -2\pi E_S k^{-3}\mathrm{Im}\left[G_{ij}(\mathbf{x}_\mathrm{A},\mathbf{x}_\mathrm{B},\omega)\right] \quad (1)$$

where $\mathbf{x}_\mathrm{A}$ and $\mathbf{x}_\mathrm{B}$ are position vectors, ω is circular frequency, u_i is displacement in direction i, * means complex conjugate, angular brackets ($\langle\ \ \rangle$) denote azimuthal average, $E_S = \rho\omega^2 S^2$ is energy density of S waves, $k = \omega/\beta$ is wavenumber of S waves, β is S wave velocity, S^2 is average spectral density of S waves. Green's function $G_{ij}(\mathbf{x}_\mathrm{A},\mathbf{x}_\mathrm{B},\omega)$ is displacement at $\mathbf{x}_\mathrm{A}$ in direction i produced by a unit load at $\mathbf{x}_\mathrm{B}$ in direction j (Sánchez-Sesma *et al.*, 2011a). In order to calculate the theoretical energy density at a given point $\mathbf{x}_\mathrm{A}$, we rewrite Eq. 1 assuming $\mathbf{x}_\mathrm{A} = \mathbf{x}_\mathrm{B}$ as:

$$E(\mathbf{x}_\mathrm{A}) = \rho\omega^2\langle u_m(\mathbf{x}_\mathrm{A})u^*{}_m(\mathbf{x}_\mathrm{A})\rangle = -2\pi\mu E_S k^{-1}\mathrm{Im}[G_{mm}(\mathbf{x}_\mathrm{A},\mathbf{x}_\mathrm{A})] \quad (2)$$

where μ is shear modulus. Equation 2 is valid even if the summation convention is ignored (Perton *et al.*, 2009), so $E(\mathbf{x}_\mathrm{A})$ can be written as $E_m(\mathbf{x}_\mathrm{A})$, which is directional energy density (DED) along direction m.

We may express the H/V spectral ratio in terms of energy densities. For instance, $E_1(\mathbf{x},\ \omega)$ is proportional to $\langle {u_1}^2\rangle = \langle {H_1}^2\rangle$. It is common to eliminate the angular brackets while writing the expression for the average H/V spectral ratio as:

$$\frac{H(\omega)}{V(\omega)} = \sqrt{\frac{E_1(\mathbf{x},\omega)+E_2(\mathbf{x},\omega)}{E_3(\mathbf{x},\omega)}} \quad (3)$$

where E_1, E_2 and E_3 are the energy densities and the subscripts 1 and 2 refer to horizontal and 3 to vertical degrees of freedom. Equation 3 is the form adopted by Arai and Tokimatsu (2004).

If we assume that microtremors constitute a diffuse field, the average spectral densities may be regarded as DEDs and then we may invoke the connection between the normalized averages of energy densities of diffuse field with the imaginary part of Green's function at the source (Sánchez-Sesma *et al.*, 2011). In fact, from Eqs. 2 and 3 we can write:

$$\frac{H(\omega)}{V(\omega)} = \sqrt{\frac{\mathrm{Im}[G_{11}(\mathbf{x},\mathbf{x};\omega)]+\mathrm{Im}[G_{22}(\mathbf{x},\mathbf{x};\omega)]}{\mathrm{Im}[G_{33}(\mathbf{x},\mathbf{x};\omega)]}} \quad (4)$$

This equation (Sánchez-Sesma *et al.*, 2011) shows average microtremor H/V spectral ratio corresponds to the square root of the ratio of the imaginary part of horizontal displacement for a horizontally applied unit harmonic load, $\mathrm{Im}[G_{11}]$ and $\mathrm{Im}[G_{22}]$, and the imaginary part of vertical displacement for a vertically applied unit load, $\mathrm{Im}[G_{33}]$

1D Analysis Considering the Velocity Model of Uji Campus

In order to check the validity of the theory, we calculated theoretical H/V spectral ratio based on our theory assuming a 1D structure at one of the microtremor measurement sites in Uji campus, site C. Figure 6 shows the velocity profile obtained from VSP survey near site C (Iwata *et al.*, 2001). We assumed a simple 3 layered 1D velocity model as shown in Table 1 denoted by thick lines in Fig. 6 according to the results of the VSP survey. Red and blue lines are the P wave and S wave velocity, respectively. The theoretical H/V spectral ratio calculated from the velocity model is shown in Fig. 7 with red and blue lines. In the figure, we also plot the the observed microtremor H/V spectral ratio (black line) as well as the H/V spectral ratio based on ellipticity of Rayleigh waves (gray line). We can see that the theoretical H/V spectral ratio fit that of the observed for NS/UD component which can be interpreted as 1D structure. Also, the peak and the dip frequency of the H/V spectral ratio derived from the ellipticity of Rayleigh waves match both the theory and observation.

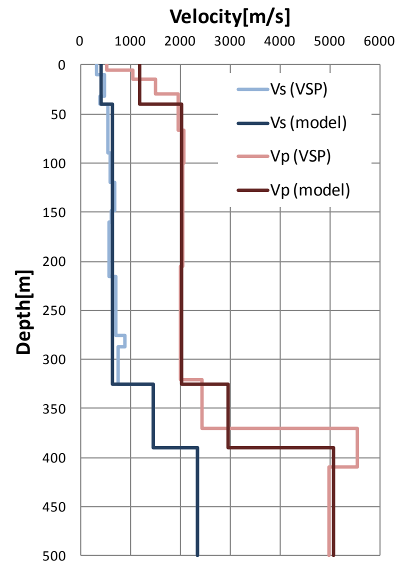


Fig. 6. Velocity profile from VSP survey near site C and assumed 1D velocity model with 3 layers. Red and blue lines are the P wave and S wave velocity, respectively.

Table 1. Three-layered 1D velocity model at site C

	Thickness [m]	Vp [m/s]	Vs [m/s]	Density [kg/m^3]	Q
1	40	1,177	406	1,900	100
2	285	2,209	638	1,900	100
3	65	2,950	1,450	2,100	100
bedrock	-	5,083	2,348	2,600	100

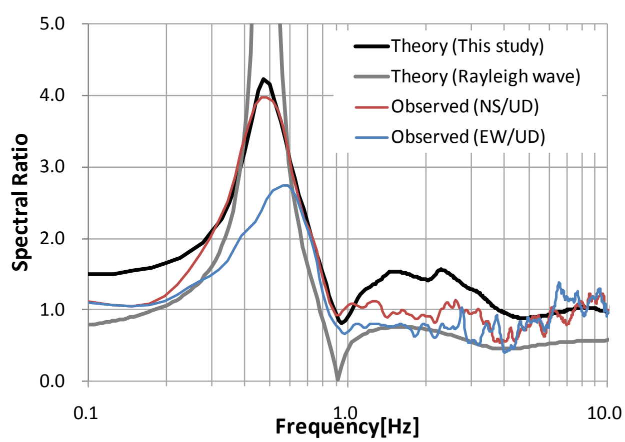


Fig. 7. Theoretical H/V spectral ratio (black line) compared against observed H/V spectral ratio of June 21st, 2010 (red and blue lines) and H/V spectral ratio derived from ellipticity of Rayleigh waves (gray line). Red and blue lines are of NS/UD and EW/UD components the observed H/V spectral ratio, respectively.

NUMERICAL 3D H/V SPECTRAL RATIO

In order to take in account of the 2D or 3D subsurface structure, we need to incorporate a numerical approach. In this study we use 3D staggered-grid FDM (Graves, 1996) to study cases of laterally heterogeneous elastic layers over a half-space.

Layered 1D Velocity Model

For calibration purposes, we first checked that numerical calculations by FDM give the same results as theoretical ones. We assume a flat-layered model with a single layer over half-space, equivalent to site C. The condition for 3D FDM calculation is listed in Table 2 and the velocity model for the flat-layered case is listed in Table 3. Figure 8a display the schematic image of the velocity model of the one layer 1D model. We apply a unit load to the surface of the model at the target position and retrieve the imaginary part of the Green's function from the response at the same position.

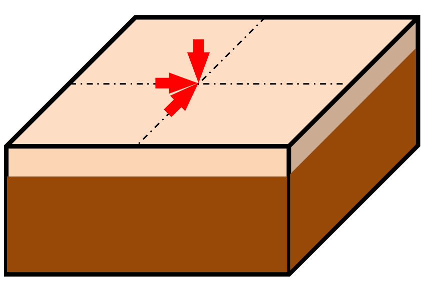
Figure 9 shows the comparison of theoretical and numerical results for 1D velocity model shown in Table 3 and Fig. 8a. Figure 9a is the imaginary part of Green's function for horizontal (dark) and vertical (light) components. The blue and orange lines are for 3D FDM and theoretical results, respectively. The FDM results shows fairly good match to the theoretical one. Figure 8b is the H/V spectral ratio derived from the imaginary part of Green's function shown in Fig. 8a. Since the imaginary part of Green's function match each other, the H/V spectral ratio also shows good match, naturally.

Table 2. Conditions of 3D FDM calculation

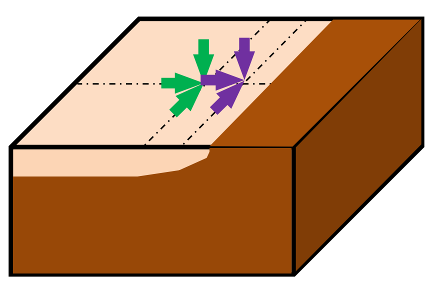
Parameter	Order of spatial accuracy	Model size	Grid spacing	Total grids	Low pass filter	Time increment	Time step	Total duration	Absorbing boundary
Value	4	Length 4 km Width 4 km Depth 2 km	10 m	32 million	13.0 Hz	0.007 sec.	30000	21.0 sec.	25 grids

Table 3. One layer 1D velocity model equivalent of site C for testing FDM calculation

	Thickness [m]	P wave velocity [m/s]	S wave velocity [m/s]	Density [kg/m^3]	Q
1	390	1,985	661	1,930	9999
bedrock	-	5,083	2,348	2,600	9999

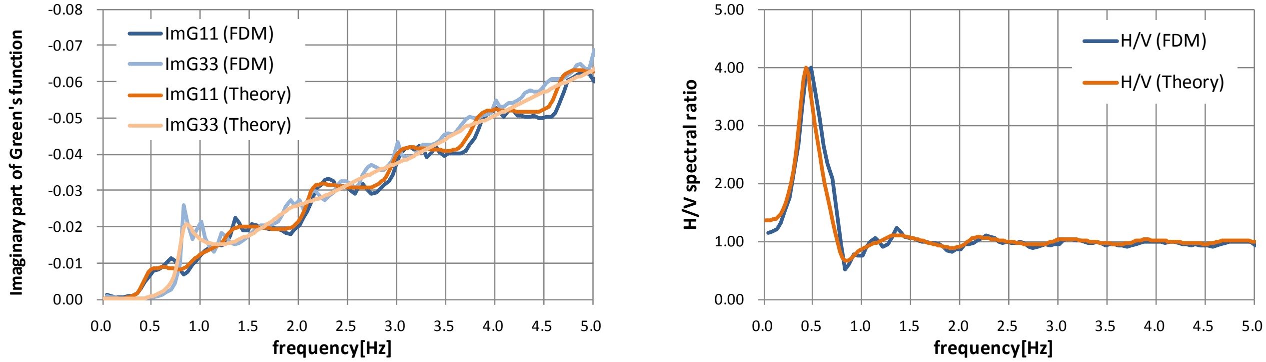


(a) One layer 1D velocity model



(b) One layer 2D velocity model

Fig. 8. Schematic image of (a) One layer 1D velocity model and (b) One layer 2D model, used for 3D FDM calculation. The colored arrows show the direction of the unit load.



(a) Imaginary part of Green's function (b) H/V spectral ratio

Fig. 9. Comparison of (a) imaginary part of Green's function and (b) H/V spectral ratio of theoretical and numerical results for 1D velocity model in Table 3. The two calculations show good match.

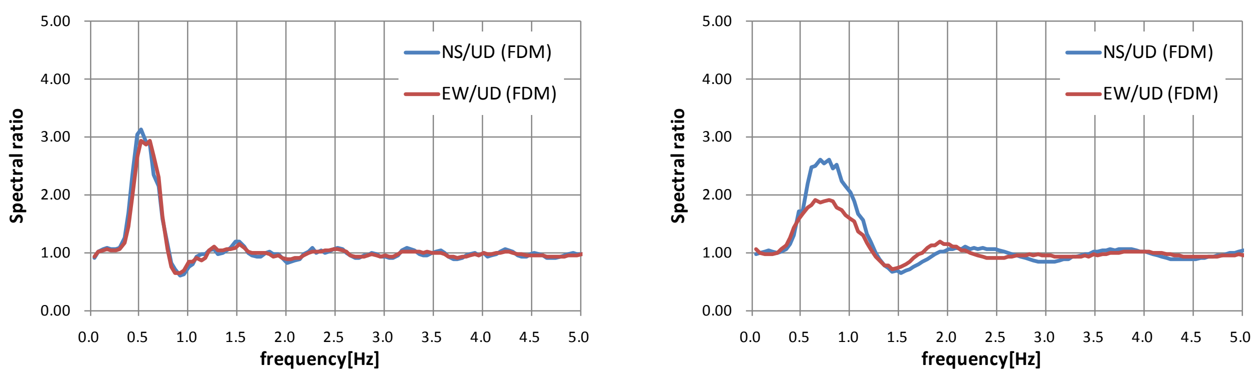
<u>Layered 2D Velocity Model</u>

Now we consider a laterally heterogeneous velocity structure. We use a 2D velocity model and calculate the 3D wave field. The 2D velocity model consists of l layer over half-space that gradually gets shallow from west to east and eventually pinches out at 1,200m (120grids) from the boundary of the model as shown in Fig. 8b. The maximum depth of the 2D basin model is 420m. The parameters of the model are listed in Table 4. For this model, we first calculated the H/V spectral ratio at position of site C, which corresponds to the green arrow in Fig. 8b. The H/V spectral ratio for this case is shown in Fig. 10a. The H/V spectral ratio for NS/UD and EW/UD components do not have large difference as expected from the observed H/V spectral ratio at site C.

Next, we did the same calculation except we moved the target position 620m (62 grids) to the east, closer to the basin edge as shown by the purple arrows in Fig 8b. The H/V spectral ratio for this case is shown in Fig. 10b. We can see that for this case the NS/UD and EW/UD components have large difference corresponding to the difference seen in the observed H/V spectral ratio.

Table 4. One layer 2D velocity model.

	Thickness [m]	P wave velocity [m/s]	S wave velocity [m/s]	Density [kg/m^3]	Q
1	< 420	1,985	661	1,930	9999
bedrock	-	5,083	2,348	2,600	9999



(a) Site C (b) Site 620m east of site C (closer to the edge)

Fig. 9. Comparison of H/V spectral ratio of NS/UD and EW/UD at (a) site C and (b) site 620m east of site C closer to the edge, calculated by FDM for 2D basin model. There is a large difference between two components for (b).

DISSCUSSION AND CONCLUSIONS

From the H/V spectral ratios of microtremor data obtained at Uji campus of Kyoto University, Kyoto, Japan, we confirmed directional dependency that seems to correspond to the deep 2D basin structure. The NS/UD component which is parallel to the strike of Obaku fault, has larger peak and lower peak frequency compared to the EW/UD component, which is perpendicular to the fault trace and the 2D basin structure. Assuming a 1- structure beneath Uji campus, the theoretical H/V spectral ratio fit the NS/UD component of the data. This shows that the NS/UD component acts as 1D site but on the other hand, the EW/UD component is affected by the 2D basin structure and acts differently. In order to simulate the H/V spectral ratio for site with lateral heterogeneity, we used 3D FDM to numerically calculate Green's function to derive H/V spectral ratio by the same procedure as theoretical H/V spectral ratio. From the comparison with theoretical and observational H/V spectral ratios, we confirmed the validity to use FDM to calculate H/V spectral ratio from imaginary part of Green's function for sites with lateral heterogeneity.

Although we succeeded in simulating the difference between the orthogonal two horizontal components of the microtremor H/V spectral ratio qualitatively, the H/V spectral ratios obtained using the assumed 2D basin model from FDM calculation cannot explain the observed data. This may be the indication that the actual bedrock may have steeper dip than the model used in this study, or has a shallow layer with large contrast not included in the model

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