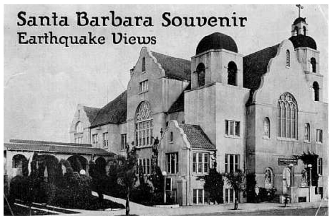

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T-F-K ANALYSIS OF SURFACE WAVES USING THE CONTINUOUS WAVELET TRANSFORM

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ABSTRACT

An alternative approach to analyze the phase velocity dispersion of surface waves from active seismic experiments is proposed. To decompose the seismic wave-field, the continuous wavelet transform is applied. This gives the possibility of accurately localize the phase information in time, and to isolate and extract the most significant contribution of surface-waves. To extract the dispersion information, then, a hybrid technique is applied to the narrow-band filtered seismic recordings. The technique combines the flexibility of the slant-stack method in identifying waves that propagate in space and time, with the resolution of f-k approaches. This is particularly beneficial for higher modes identification in case of high noise level. Estimation of the surface wave phase-delay is performed in the frequency domain by mean of a covariance matrix averaging procedure over successive wave-field excitations. Thus, no record-stacking is necessary in the time domain and a large number of consecutive shots can be used. Since the method does not require any hardware-based source triggering, it is particularly suitable for continuous recordings when using seismological equipment (e.g. together with passive acquisition of ambient vibrations).To demonstrate the effectiveness of the method we show its performance on synthetics data.

INTRODUCTION

In the paper we present an alternative way of performing active seismic acquisition, based on the continuous recordings of seismological stations (Fig. 1). Such approach arises from the practical exigence of optimizing the use of the available instruments for passive acquisition, with that of investigating shallower velocity structures. As a matter of fact, in surface wave analysis, the resolution on depth is controlled by the frequency range of investigation and the seismic velocity structure of the site (Aki and Richards, 1980). Ambient noise, in general, is suitable for the investigation at relatively low frequencies only, roughly <10~20Hz, (Horike, 1985). This is mostly because of the strong attenuation of the wave-field generated by too far and scarcely energetic sources (natural or anthropogenic). To improve the resolution on the shallower depths then, high-energy artificial sources have to be used (Park et al., 2005). This gives the possibility of exiting surface wave higher modes, which are rarely identified with passive seismic (Poggi and Fäh, 2010).

Nowadays, several techniques are available to analyze the surface-wave dispersion from simultaneous recordings. Most of them belong to the so called domain-transformation methods and consists in two main categories: the *τ-p* (e.g. McMechan and Yedlin, 1981; Xia et al., 2007) and the *f-k* transforms (e.g. Lacoss et al., 1969; Nolet and Panza, 1976), with their respective variants. All of these methods present some advantages and disadvantages, and thus it is difficult to generalize which could be the best performing at a specific site and for a specific task. For example, *τ-p* methods performs generally better than *f-k* for short-duration transient signals, but have conversely limited frequency resolution. On the other side, *f-k* is more sensitive to uncorrelated noise. This is crucial especially for the identification of surface-wave higher modes (Strobbia, 2003). Therefore, we combine the features of the two approaches into a hybrid time-frequency-wavenumber method, based on the continuous wavelet transform.

Fig. 1. Example of linear array of seismological stations during a mixed active/passive acquisition survey. Each station is an independent high-resolution recording unit.

MULTIPLE SHOT TRIGGERING OF CONTINUOUS RECORDS

Continuous recording comes from the exigence of optimizing the use of our acquisition equipment for passive seismic in combination with active experiments. The main practical disadvantage in using seismological stations stays in the difficulty in triggering the initial time (*t0*) of the artificial source. In practice, however, in all approaches based on cross-correlations (in the time or the frequency domain) the knowledge of an absolute *t0* is not necessary, since only the relative phase-delay information between traces is analyzed. For each shot, then, a simple relative reference initial time can be used instead of an absolute one.

To estimate a relative reference time of a particular shot, we implemented an automatic triggering procedure based on energy distribution localization. We first define a reference trace $u(x_{Ref}, t)$ (e.g. offset 1) for which the amplitude envelope *A* is computed as modulus of the analytical representation of the input signal. From this envelope function, all relative maxima are then picked and sorted according to their energy level:

$$A(t0, t1, ..., tn) = \left| u(x_{Ref}, t) + j\left[u(x_{Ref}, t) * \frac{1}{\pi t} \right] \right|_{Max} \qquad (1)$$

Therefore, for *n* consecutive shots performed during the survey, the time instants (*t0,t1,…,tn*) corresponding to the first *n* most energetic amplitudes are extracted (Fig. 2). With respect to a simpler approach based on amplitude picking, the advantage in using the modulus of the analytical signal stays in the minimization of the bias introduced by the random phase interaction, which might produce peaks at different relative times of the consecutive shots.

THE TIME-FREQUENCY-WAVENUMBER ANALYSIS

In the *f-k* methods like the classical beamforming (Lacoss et al., 1969), the signal's covariance matrix is obtained, at specific frequencies, from the complex conjugate cross-products between Fourier transformed signals over the different offset locations. Thus, if a single wave propagates with a certain phase velocity, the elements of the covariance matrix will provide the relative phase-delays information between all receiver pairs. If several waves propagate at the same time but with similar phase-velocities, however, such approach can hardly separate out the different phase contributions. We propose therefore a different way to estimate the covariance matrix. We make use of the time-frequency analysis using wavelet transform to account for travel-time delays induced by wave propagation over the different offsets. Basically, the covariance matrix is obtained from extracting and correlating only those values of the complex spectrogram that satisfy a specific velocity of propagation. The procedure is similar to a *τ-p* analysis, but applied here to the estimation of the instantaneous phases.

For a given frequency and velocity, then, the single elements of this covariance matrix can be phase re-corrected to a common (and relative) reference time, according to the relative travel-time delays. In case of multiple shots, moreover, successive covariance matrices can be stacked and averaged, to enhance the phase-delay estimation with respect to background uncorrelated noise. Finally, the *f-k* power spectrum can be computed, using either the beamforming technique (Lacoss et al., 1969) or any other high-resolution method, based e.g. on data weighting (e.g. Burg, 1967; Capon, 1969) or signal eigen-decomposition (e.g. Schmidt, 1986).

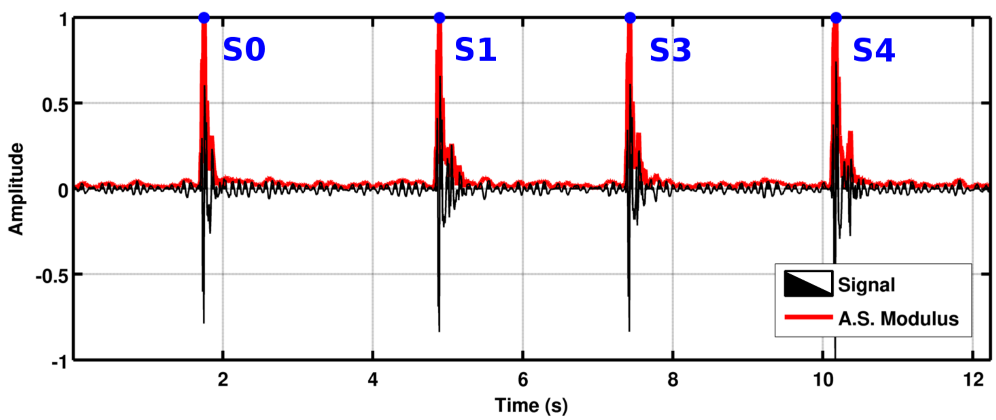


Fig. 2. Automatic triggering of the reference time of multiple shots using the modulus amplitude of the analytical signal (A.S., in red). The obtained t0 is the relative reference time (t-ref) for phase correction, however does not represent the true shot time.

Travel-time covariance matrix estimation

In all beamforming type techniques (classical or high-resolution), the signal covariance (or cross-correlation) matrix has to be computed over the different discrete frequencies to estimate the *f-k* power spectrum. Thus, a Fourier analysis of the recordings is required. Usually, the whole record window is used for the computation of the Fourier spectra. In such way, however, the influence of body waves and other contributions (e.g. noise, air blast) can significantly affect the final result, introducing some bias on the phase estimate of surface waves. To avoid this problem, it is common procedure to manually select some appropriate windows (tapering in the time-offset domain) to exclude the direct and refracted arrivals and emphasize therefore the surface wave content. Such approach, however, is influenced by the subjectivity of the operator who defines the window, since no strict rules are (and can be) established for this procedure. In some cases, portions of the traces cut out by manual windowing can still contain usable surface wave information. Moreover, the length of the selected windows can be very different at different offsets, affecting the robustness of the covariance matrix elements estimate.

To overcome this problem, we perform a time-frequency analysis of the records, using the continuous wavelet transform (Fig. 3). Such approach has the advantage to make the phase information separable in time (as instantaneous estimation) and thus for the different wave contributions. Therefore, once a propagating wave is identified, its instantaneous phase can be extracted at a specific frequency. Clearly, the quality of the result is controlled by the trade-off between the resolution in time and frequency of the wavelet transform. In general, the higher is the resolution in time, the lower is in frequency and vice versa (following the Heisenberg uncertainty principle).

The problem is therefore how to automatically isolate the propagation of a particular wave on the complex spectrogram obtained from the wavelet decomposition. For surface waves this cannot be simply done by travel-time picking, since the surface-wave arrival-time cannot be localized because of the dispersion. To solve this, we implemented a direct search approach, based on the idea of *τ-p* analysis (or the slant-stack), but applied here to wavelet filtered signals. If we define the wavelet-transform *w* of the signal *u* at a specific frequency *f* and offset *x* as:

$$w(f,x,t)=\int_{-\infty}^{\infty}u(x,\tau)w_m^h(f,t,\tau)d\tau \tag{2}$$

where w_m is the filter bank base to be used (or the mother wavelet), then the offset-vector obtained by those complex values that satisfy a specific velocity of propagation *v* and source delay-time *t* at the different offsets can now be written as:

$$\boldsymbol{S}(f,t,v)=\left[w\left(f,x_1,t+\frac{(x_1-x_{Ref})}{v}\right),\ldots,w\left(f,x_n,t+\frac{(x_n-x_{Ref})}{v}\right)\right]^T \tag{3}$$

Therefore, the covariance matrix is:

$$\hat{\boldsymbol{R}}(f,t,v)=E\left\{\boldsymbol{S}(f,t,v)\cdot\boldsymbol{S}(f,t,v)^h\right\} \tag{4}$$

In practice, $\hat{\boldsymbol{R}}(f,t,v)$ is computed as the Hermitian cross-products between those instants of the complex spectrograms that correspond to a specific wave (see examples in Fig. 4). With respect to the classic way of estimating the covariance matrix - using Fourier transform - the correlation depends now on three independent variables and thus is *travel-time dependent*. However, since the *t* and *v* parameter pairs are not known a-priori, the matrix has to be recomputed for any possible combination, within a given reasonable range of expectation. The delay-time search parameter *t*, in particular, is necessary because we might expect the surface waves not to develop immediately at the shot time, and higher modes not to be simultaneous with the fundamental.

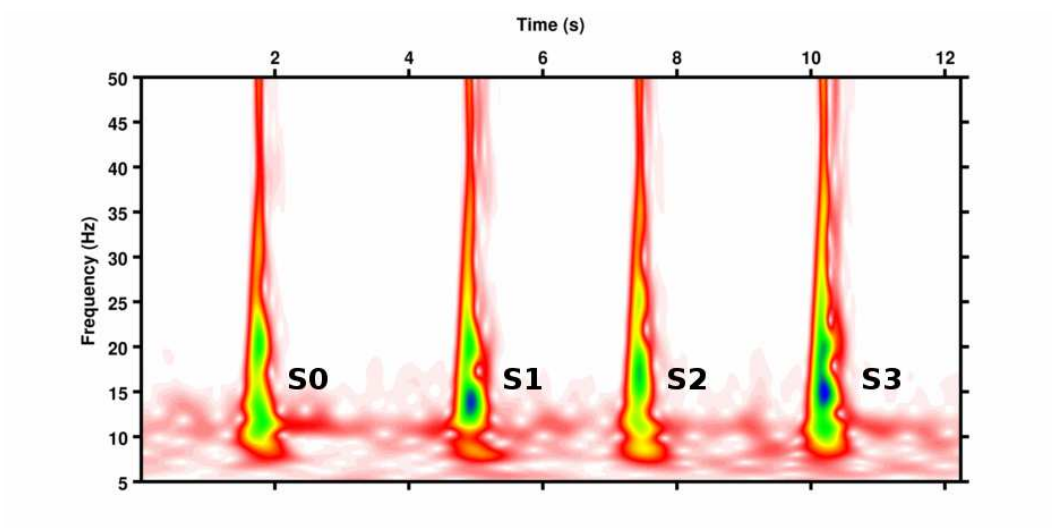


Fig. 3. Wavelet spectrogram (absolute value) of the four shots recorded at the first receiver location.

As last step, then, each element of $\hat{\boldsymbol{R}}(f,t,v)$ can be phase-corrected back to a reference time common to all traces (e.g. *t-ref* obtained by triggering), to compensate the effect of travel-time delay over the different offsets:

$$\hat{\boldsymbol{R}}_{i,j}^{\phi}(f,t_{Ref},v)=\hat{\boldsymbol{R}}_{i,j}(f,t,v)\cdot e^{\left[-2\pi f\left(t_{Ref}+\frac{(x_j-x_i)}{v}\right)\right]} \tag{5}$$

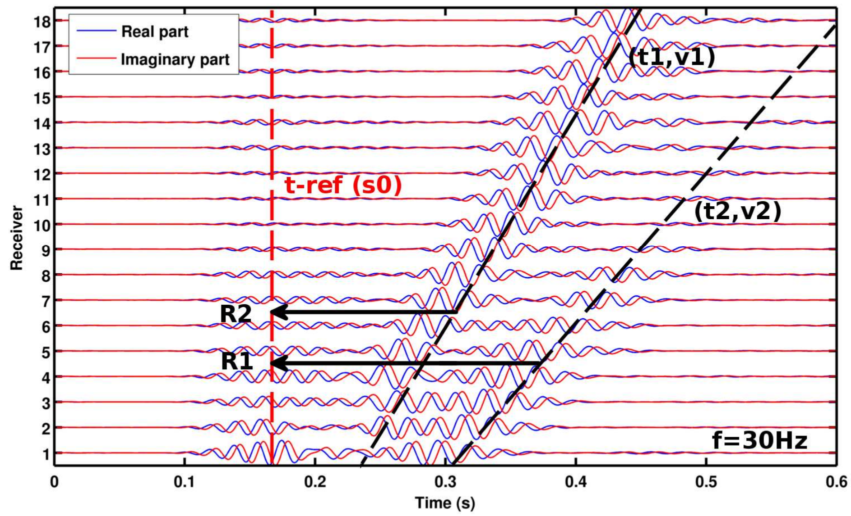


Fig. 4. Wavelet transformed traces (real and imaginary part) of a 18 stations seismic record (here filtered at 30Hz). It is possible to follow the single components propagating with different source delay-time and phase velocity (t, v). The triggered reference time (t-ref) is used for travel-time correction of the covariance matrix elements (R) for the investigated phase velocity.

Covariance matrix stacking and phase-averaging

The classic acquisition protocol for active seismic includes the stack of consecutive recordings to improve the signal-to-noise ratio. This is particularly suitable for reflection/refraction seismic, where only the correct identification of the travel-times is of primary importance, but it might not be strictly necessary for surface wave analysis and in case of continuous recordings. Averaging the phase over different shots implies the assumption that each wave excitation will produce exactly the same phase spectrum at the source. This assumption, however, might not be perfectly fulfilled in reality when simple artificial sources are used, like mini-gun or the sledge hammer. For this kind of devices it is indeed difficult to ensure that they will always operate in a repeatable fashion. This is particularly evident at rather high frequency ranges, where we can observe phase cancellation.

To enhance the final resolution of the *f-k* estimate, instead of averaging the single phase estimates, we average the phase-differences between receiver pairs. This can be done by stacking the travel-time corrected covariance matrix over consecutive *N* shots:

$$\left[\hat{\boldsymbol{R}}^{\phi}(f,t,v)\right]_{TOT} = \sum_{m=1}^{N(shots)} \frac{\left[\hat{\boldsymbol{R}}^{\phi}(f,t,v)\right]_{m}}{N} \qquad (6)$$

Such procedure minimizes the effect of uncorrelated noise, enhances the phase-delay estimation and stabilizes the covariance matrix in the use with high resolution *f-k* algorithms based on eigen-decomposition.

Finally, to account for geometrical and intrinsic attenuation, which induces wave amplitude to decrease with increasing distance to the source, a normalization procedure is applied. We use the approach proposed by Capon (1969), which consists in directly normalizing the signal covariance matrix, so that the normalization is based on the relative amplitude of each pair of cross-correlated signals:

$$\hat{\boldsymbol{R}}^{\phi}_{i,j}(f,t,v) = \frac{\hat{\boldsymbol{R}}^{\phi}_{i,j}(f,t,v)}{\left[\hat{\boldsymbol{R}}^{\phi}_{i,i}(f,t,v)\hat{\boldsymbol{R}}^{\phi}_{j,j}(f,t,v)\right]^{\frac{1}{2}}} \qquad (7)$$

for $i \neq j$, and:

$$\hat{\boldsymbol{R}}^{\phi}_{i,j}(f,t,v) = 1 \qquad (8)$$

for $i = j$.

The *f-v-t* power spectrum and grid search

Applying the procedures described above, we obtained an estimation of the signal covariance matrix that depends, other than on frequency, on the analyzed phase-velocity and source time of surface-waves. The *f-v-t* power spectrum, then, can directly be computed using the classic tools for *f-k* analysis, e.g. like classic beamforming:

$$P(f,t,v) = \frac{\boldsymbol{e}(f,v)^{\dagger}\,\hat{\boldsymbol{R}}^{\phi}(f,t,v)\,\boldsymbol{e}(f,v)}{N^2} \qquad (9)$$

From the implementation point of view, identifying and extracting the surface dispersion curves is done by means of a power-spectrum local maxima search over a three-dimensional parameter space. For simplicity, we first perform a grid search over *t*, fixed *v* and *f*. The procedure is then repeated for all combinations of *v* and *f*, respectively. Obviously, such processing is computationally more expensive than the classic approaches for surface wave analysis. However, the entire procedure can be conducted automatically and, even for a high number of shots, without the supervision of the user.

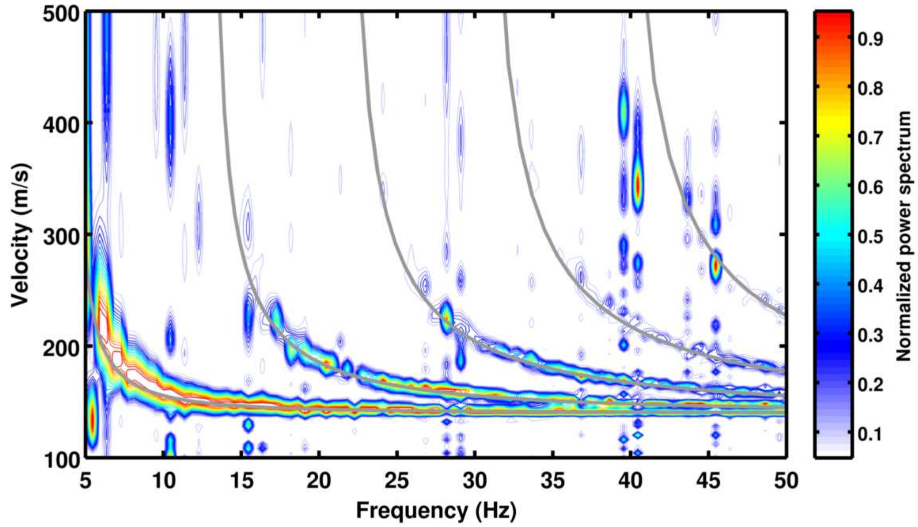
TESTING THE SYNTHETIC DATASET

A set of synthetic seismograms were generated by Sabine Latzel for a previous active seismic study (Schuler, 2008). For the modeling, an algorithm originally written by Friederich and Dalkolmo (1995) was used. The model, taken from literature (Dal Moro et al., 2006), consists of three horizontal layers with seismic velocities (Vs and Vp) progressively increasing with depth (Table 1). A point (delta) source located at the surface was employed, with frequency bandwidth of 1-60Hz. Each synthetic consists in the continuous recording of 4 consecutive shots, spaced about 2 seconds each. To emulate realistic field conditions, an amount of white (uncorrelated) noise was applied to the traces before processing. Synthetics seismograms were generated for 40 receivers locations with 2.5*m* spacing.

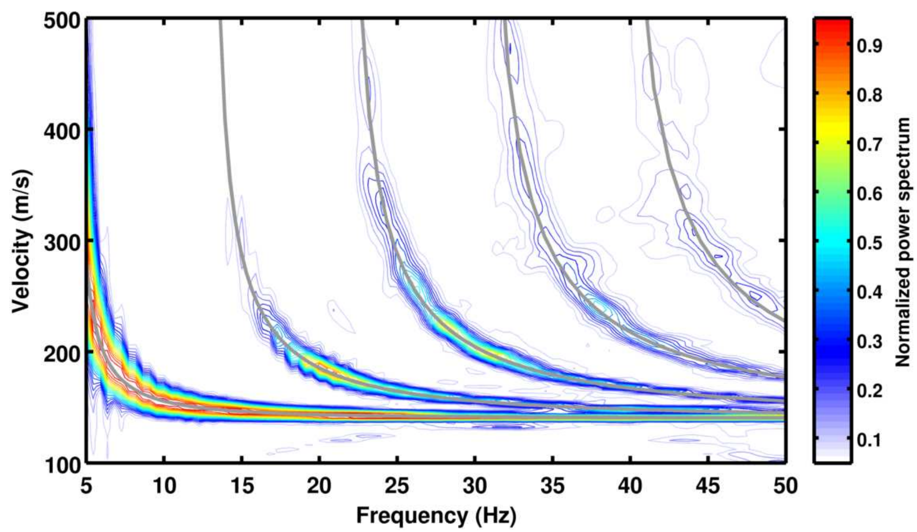
Table 1. Parameters of the one-dimensional models employed to generate active seismic synthetics.

	Thickness (m)	*Vp (m/s)*	*Vs (m/s)*	*Density (Kg/m^3)*
Layer 1	8	340	140	1700
Layer 2	8	2770	1570	2050
Layer 3	–	5200	3000	2400

For comparison, we processed the recordings using both the classic Fourier-based method and the wavelet approach (Fig. 5) using beamforming. In particular, the two methods always provide comparable results in the case of single shot and in absence of noise disturbances. However, when multiple consecutive shots are used and Gaussian noise applied, the wavelet method produces more stable results (Fig. 5, b) in comparison to the classic approach (Fig. 5, a). In more detail, the higher modes are emphasized, even at high velocities, and therefore better identifiable (Fig. 6). This can be explained by the more robust estimation of phase-delays obtained with the *t-v-f* grid search combined with the previously discussed procedures of covariance matrix stacking and normalization, in presence of strong uncorrelated noise.



(a)



(b)

Fig. 5. Frequency-velocity power spectrum of the synthetic dataset using the classic Fourier-based beamforming (a) and the wavelet method (b). In this example, the transversal (SH) component is shown, using 40 receivers with 2.5m inter-distance. Four shots were performed and a strong amount of noise applied to the traces. Here, other than the fundamental mode of Love waves, additional four higher modes can be clearly identifiable (analytical solution in gray solid line).

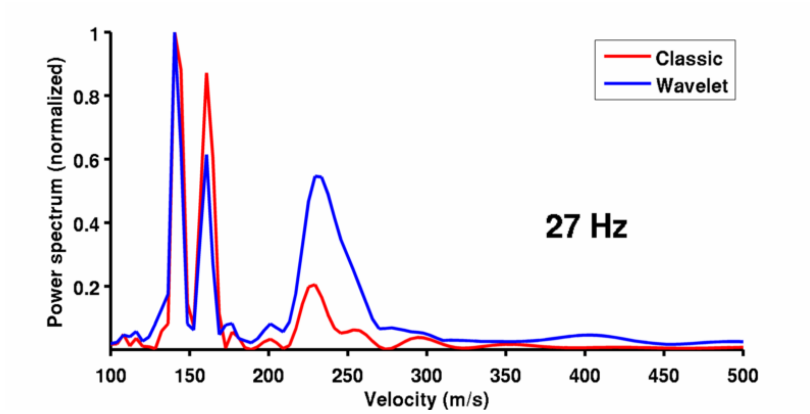


Fig. 6. Comparison of the power-spectrum computed with different approaches. Notice how the proposed wavelet-based method emphasizes the energy content of the higher modes (here of the transversal component), making them more visible (e.g. the second higher mode at 230m/s).

DISCUSSION AND CONCLUSIONS

In this paper we presented a new approach to surface wave analysis of active seismic experiments, based on the continuous recordings with seismological stations. The method is therefore suitable in combination with passive seismic acquisition. We use the continuous wavelet transform to extend the capability of beamforming techniques in detecting short transients that propagate in space with specific phase velocities. In practice, the proposed approach relies on the estimation of surface-wave travel-times to enhance the estimation of the signal covariance matrix. Moreover, stacking the covariance matrix over consecutive wave-field excitation improves the resolution on surface wave dispersion imaging in noisy environments. The method is therefore particularly advantageous for higher modes detection, that are generally more affected by the disturbance of uncorrelated noise. We tested the technique on synthetics records, where multiple modes of the surface waves were therefore detected.

As main disadvantage, however, the presented approach is computationally more expensive if compared to previous f-k methods, since it requires the re-computation of the covariance matrix for any permutation of the analyzed frequencies, phase velocities and source delay-times. Furthermore, such procedure has to be repeated for all recorded shots before stacking. Nevertheless all the search parameters are here independent, and thus the algorithm is easily parallelizable and scalable. It suits therefore for distributed computing on multi-core/multi-processor machines.

To decompose the wave-field, a time-frequency-wavenumber analysis based on the wavelet-transform has been applied to the active seismic records. With few modifications, we are confident that the presented method might also be successfully extended to ambient noise processing, to enhance the capability of separating out the different wave contributions (e.g. body and surface waves). As future development, moreover, we plan to extract and analyze, other than dispersion, the surface wave amplitude information from *t-f-k* power spectrum estimates. On three-component recordings, for example, this might allow the estimation of the Rayleigh wave ellipticity function in active seismic experiments.

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