Nonlinearity in Soil Response: Nonlinear Volumetric Mechanism

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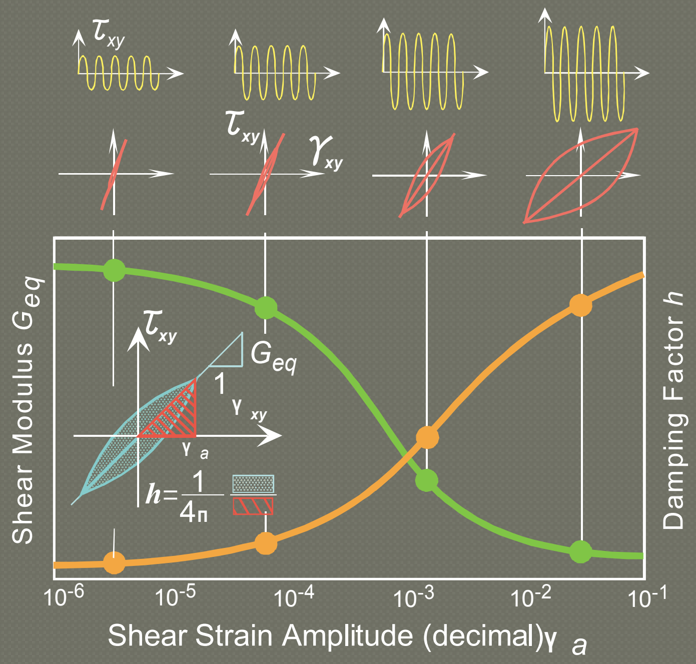
Soil non-linearity: Isotropic (equivalent) linear materials



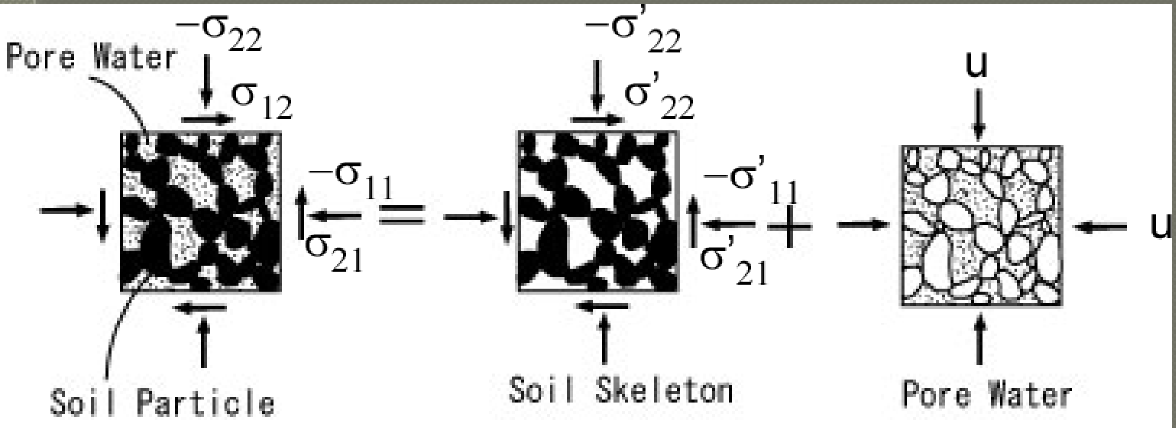
$$\boldsymbol{\sigma} = -p\mathbf{I} + \mathbf{q}$$

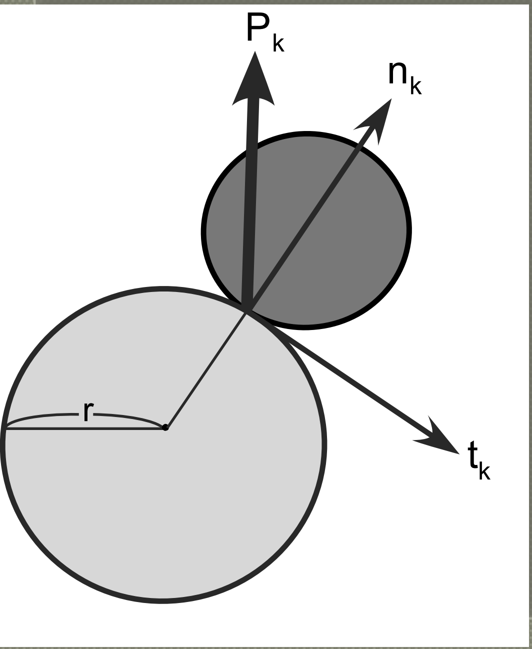
$$p = -K\mathbf{I} : \boldsymbol{\varepsilon}$$

$$\mathbf{q} = 2G\left(\boldsymbol{\varepsilon} - \frac{1}{3}\mathbf{I} \otimes \mathbf{I} : \boldsymbol{\varepsilon}\right)$$

Seed and Idriss (1970)

Soil non-linearity
Granular materials





$$\boldsymbol{\sigma} = \boldsymbol{\sigma}' - u\mathbf{I}$$

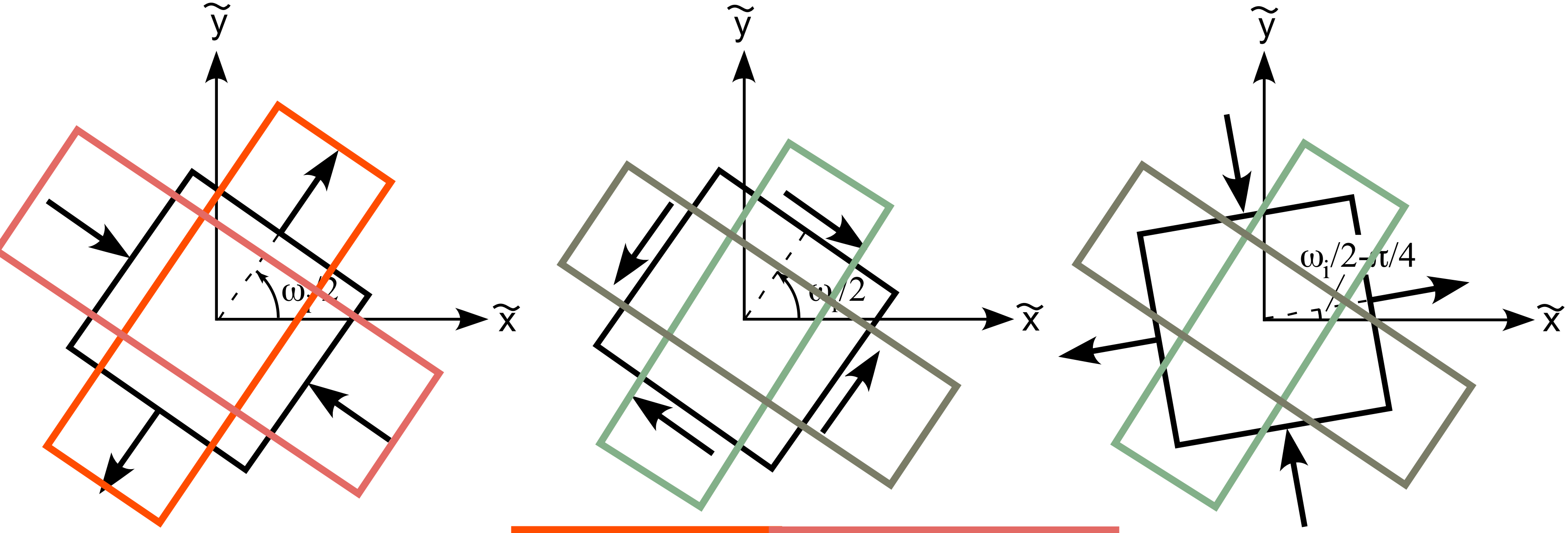
Terzaghi (1943)

$$\mathbf{P} = F\mathbf{n} + S\mathbf{t}$$

Oda (1974)

$$\boldsymbol{\sigma}' = \frac{1}{V}\sum l\left(F\mathbf{n}\otimes\mathbf{n} + S\mathbf{t}\otimes\mathbf{n}\right)$$

Virtual shear mechanism



$$\langle \mathbf{n} \otimes \mathbf{n} \rangle = \mathbf{n} \otimes \mathbf{n} - \mathbf{t} \otimes \mathbf{t}$$

$$\langle \mathbf{t} \otimes \mathbf{n} \rangle = \mathbf{t} \otimes \mathbf{n} + \mathbf{n} \otimes \mathbf{t}$$

$$\boldsymbol{\sigma}' = -p\mathbf{I} + \frac{1}{4\pi}\iint q\langle \mathbf{t} \otimes \mathbf{n} \rangle \mathrm{d}\omega \mathrm{d}\Omega$$

$$\varepsilon = \mathbf{I} : \boldsymbol{\varepsilon}, \quad \gamma = \langle \mathbf{t} \otimes \mathbf{n} \rangle : \boldsymbol{\varepsilon}, \quad \varepsilon' = \varepsilon - \varepsilon_{\mathrm{d}}$$

$$\mathrm{d}p = -K_{\mathrm{L/U}} \mathrm{d}\varepsilon'$$

$$\mathrm{d}q = G_{\mathrm{L/U}} \mathrm{d}\gamma$$

Coupling through dilatancy

Anisotropy

(Beyond equivalent linear analysis)

Iai (1993)

$$\mathrm{d}\boldsymbol{\sigma}' = \mathbf{C} : \mathrm{d}\boldsymbol{\varepsilon}'$$

$$\mathbf{C} = K_{\mathrm{L/U}} \mathbf{I} \otimes \mathbf{I} + \frac{1}{4\pi} \iint G_{\mathrm{L/U}} \langle \mathbf{t} \otimes \mathbf{n} \rangle \otimes \langle \mathbf{t} \otimes \mathbf{n} \rangle \mathrm{d}\omega \mathrm{d}\Omega$$

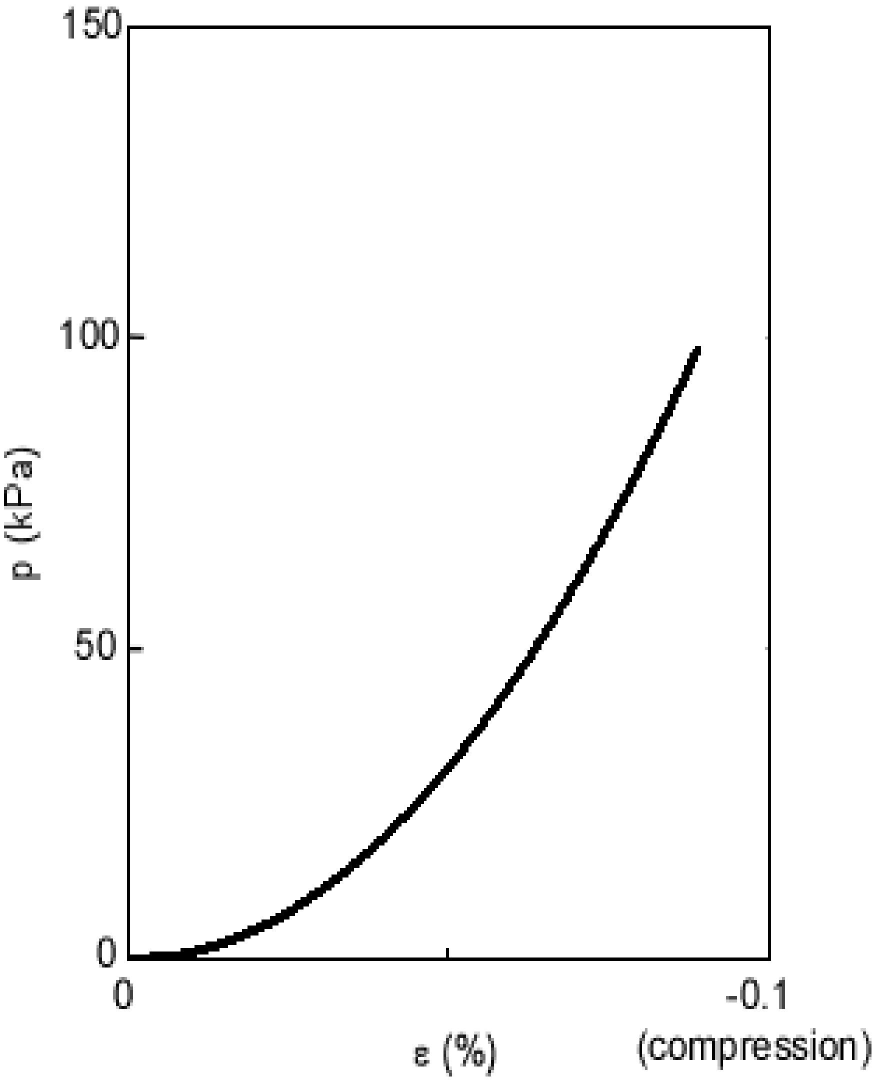
$$\mathrm{d}\boldsymbol{\varepsilon}' = \mathrm{d}\boldsymbol{\varepsilon} - \frac{1}{3} \mathrm{d}\varepsilon_{\mathrm{d}} \mathbf{I}$$

Nonlinear Volumetric Mechanism

$$K_{\mathrm{L/U}} = K_{\mathrm{a}} \left(\frac{p}{p_{\mathrm{a}}} \right)^{n_K} \quad \text{for } p \geq 0 \text{ (compression)}$$

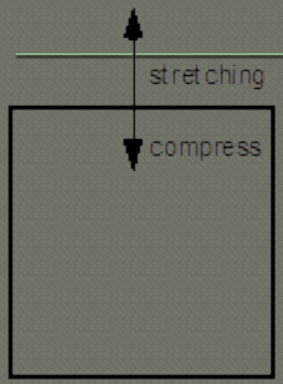
$$p = \begin{cases} p_{\mathrm{a}} \left(-\left(1 - n_K\right) \dfrac{\varepsilon'}{\varepsilon_{\mathrm{ma}}} \right)^{\frac{1}{1-n_K}} & \text{for } \varepsilon' < 0 \text{ (compression)} \\ 0 & \text{for } \varepsilon' \geq 0 \text{ (extension)} \end{cases}$$

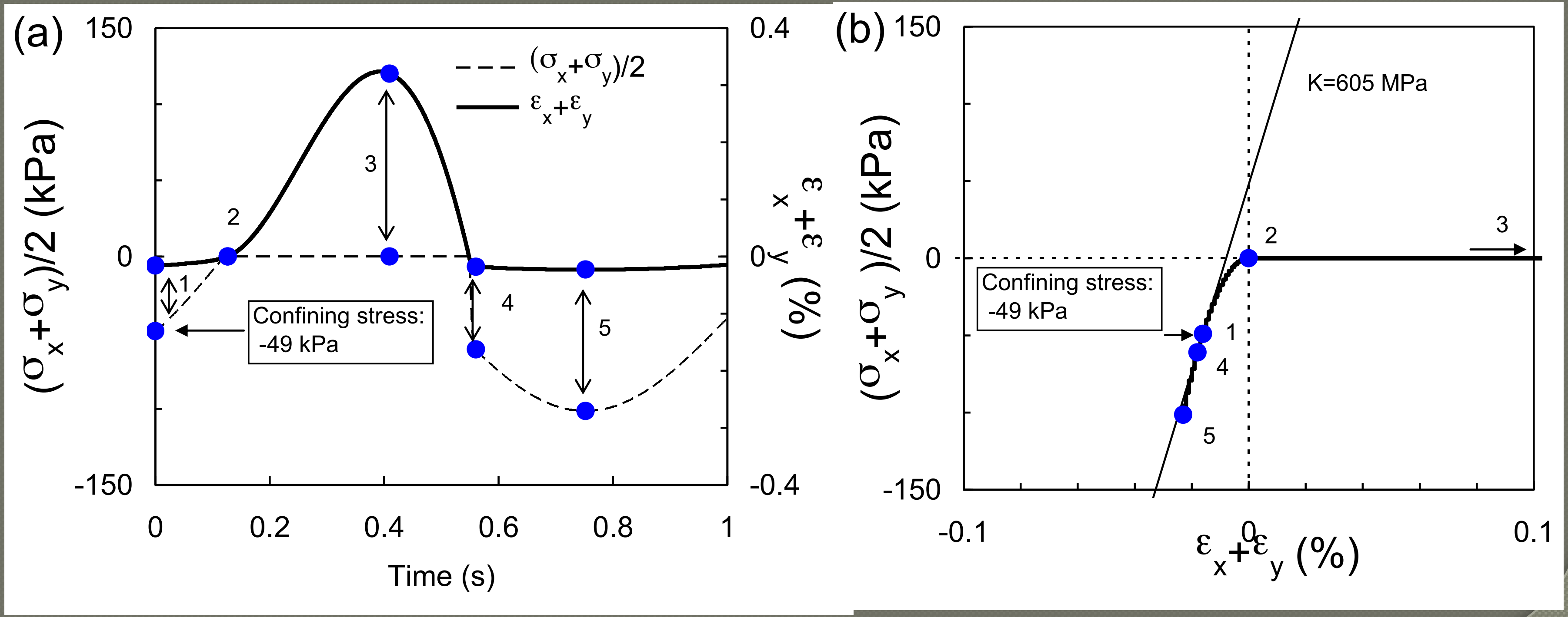
$$\varepsilon_{\mathrm{ma}} = \frac{p_{\mathrm{a}}}{K_{\mathrm{a}}}$$



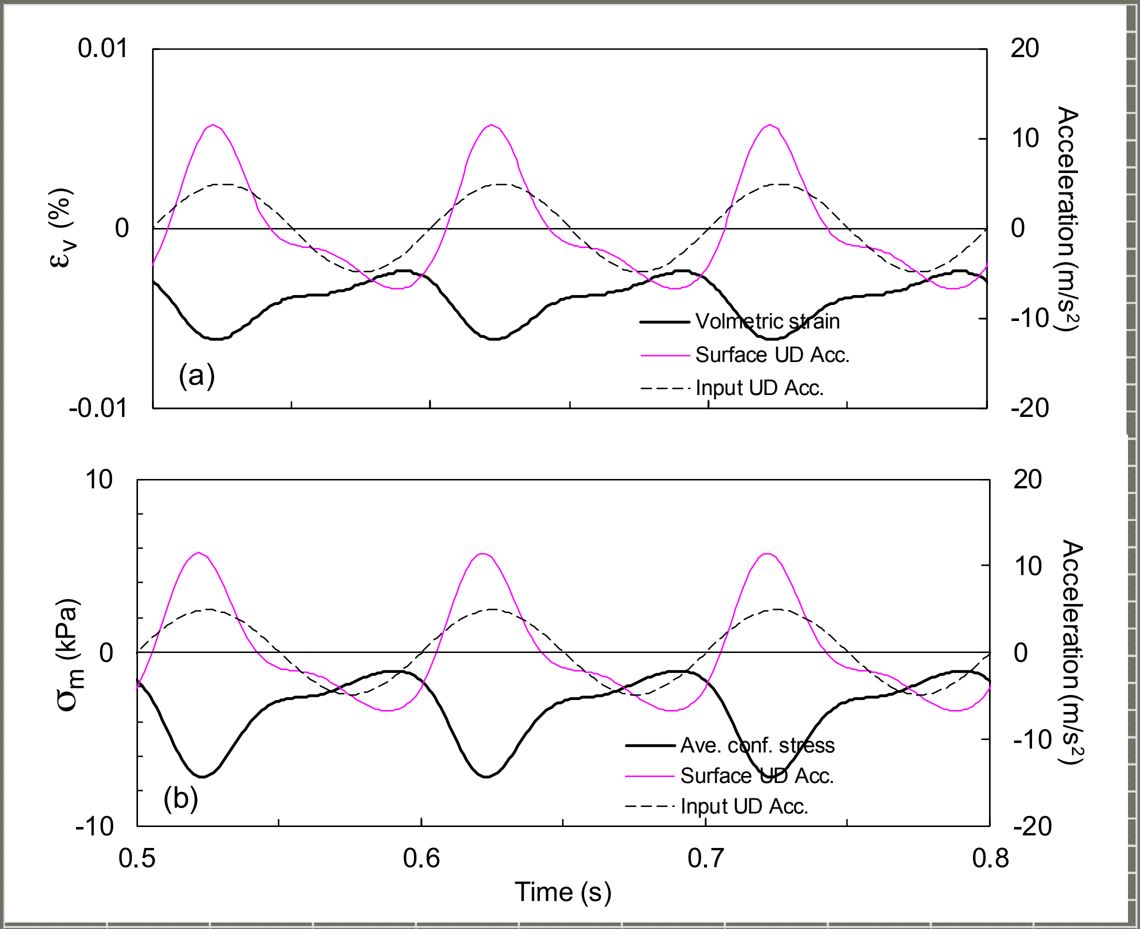
150
100
50
0
p (kPa)
0
-0.1
(compression)
ε (%)

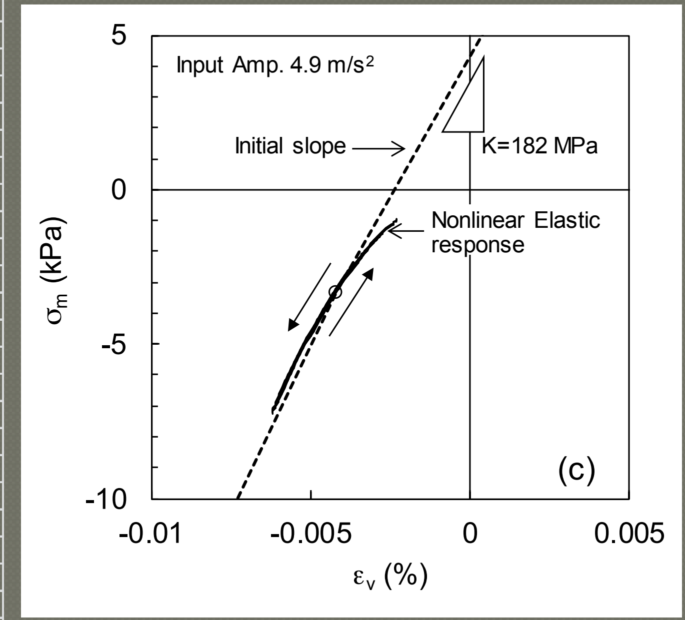
Model Behavior under Cyclic Vertical Streching and Compression



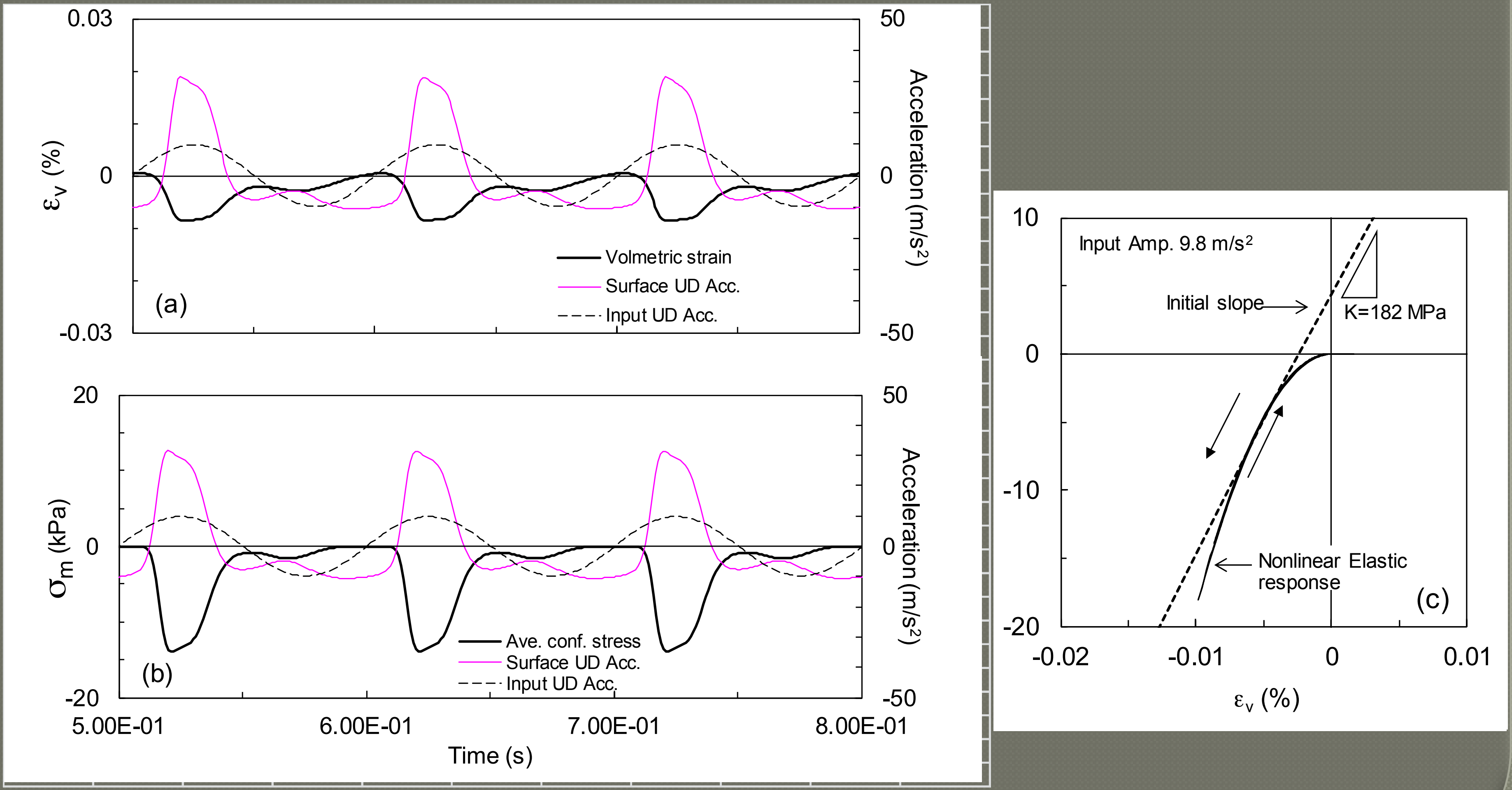


Model Behavior under cyclic Vertical Input Acceleration (0.5g)

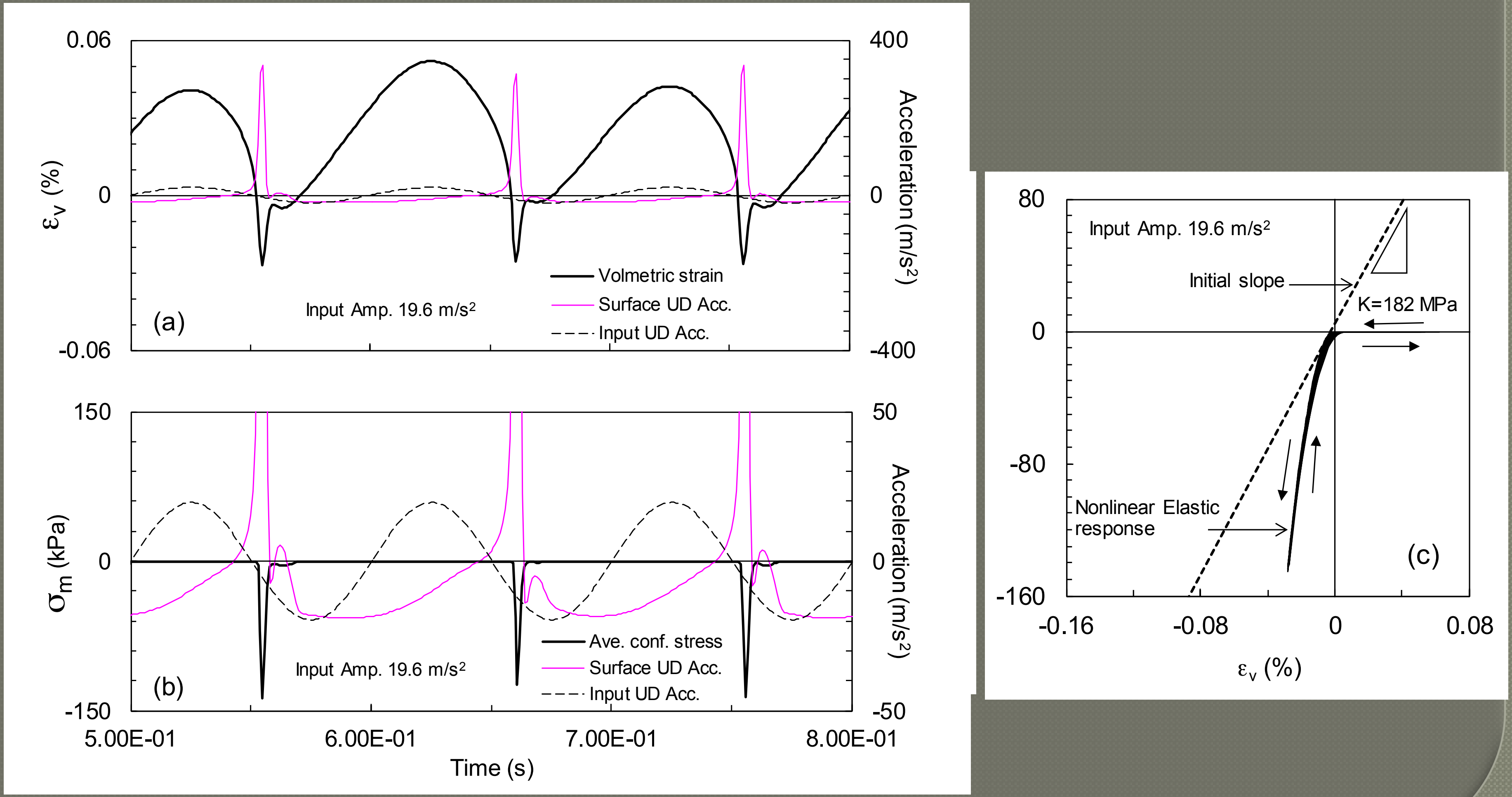




(1g)

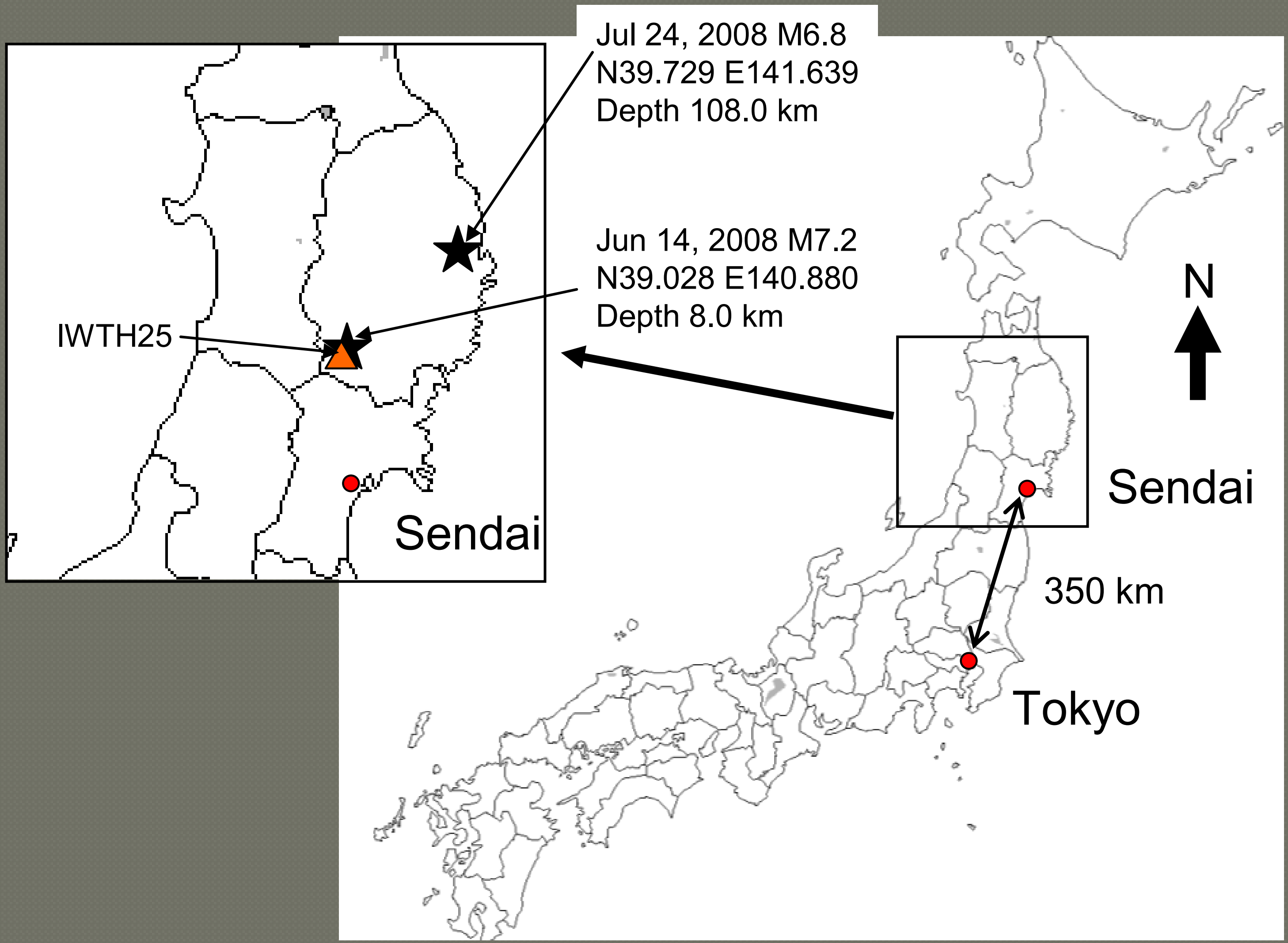

0.03
0
-0.03
ε_v (%)
50
0
-50
Acceleration (m/s²)
Volmetric strain
Surface UD Acc.
Input UD Acc.
(a)
20
0
-20
σ_m (kPa)
Ave. conf. stress
Surface UD Acc.
Input UD Acc.
(b)
5.00E-01
6.00E-01
7.00E-01
8.00E-01
Time (s)
10
0
-10
-20
Input Amp. 9.8 m/s²
Initial slope
K=182 MPa
Nonlinear Elastic
response
(c)
-0.02
-0.01
0
0.01
ε_v (%)

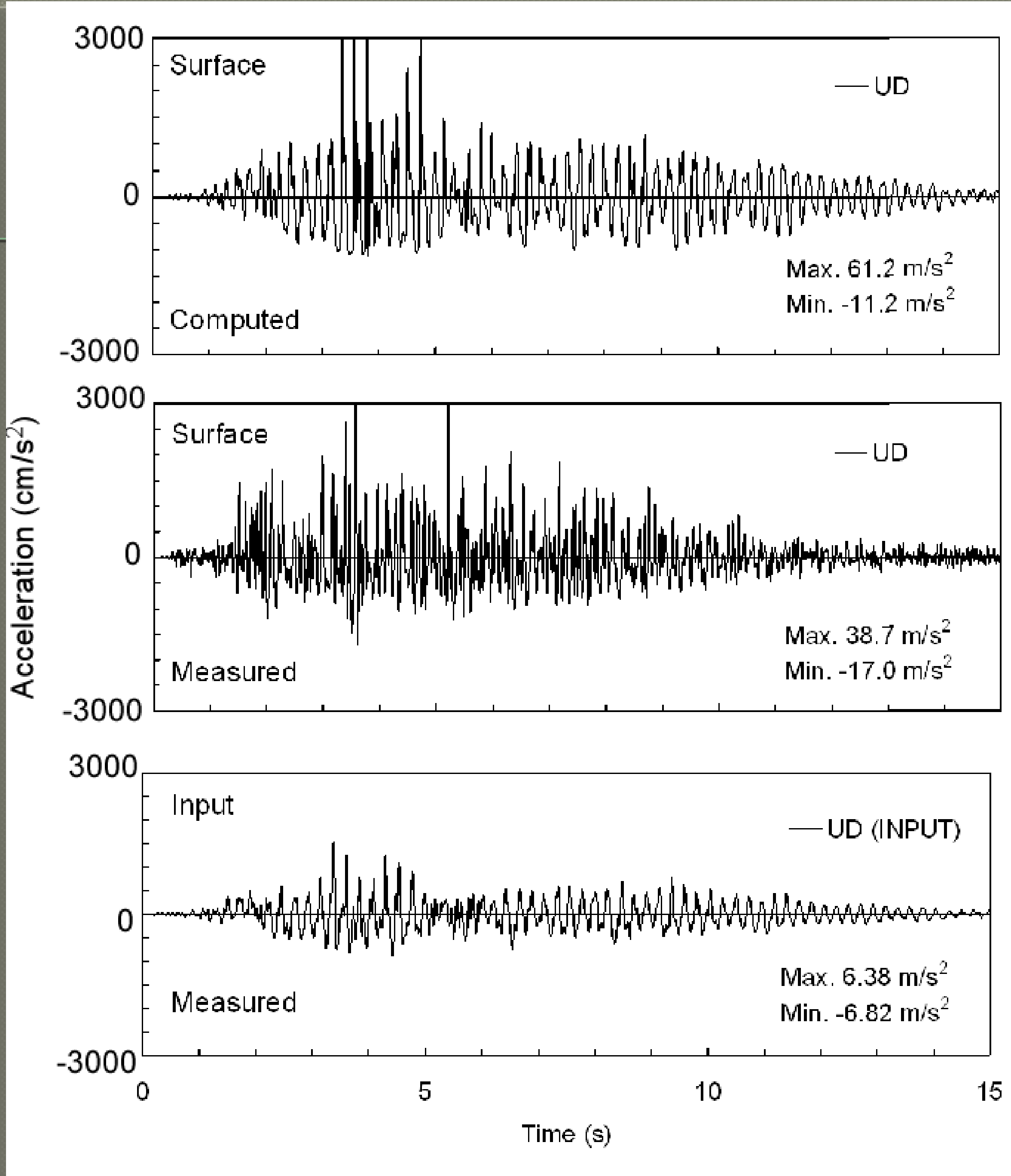
(2g)



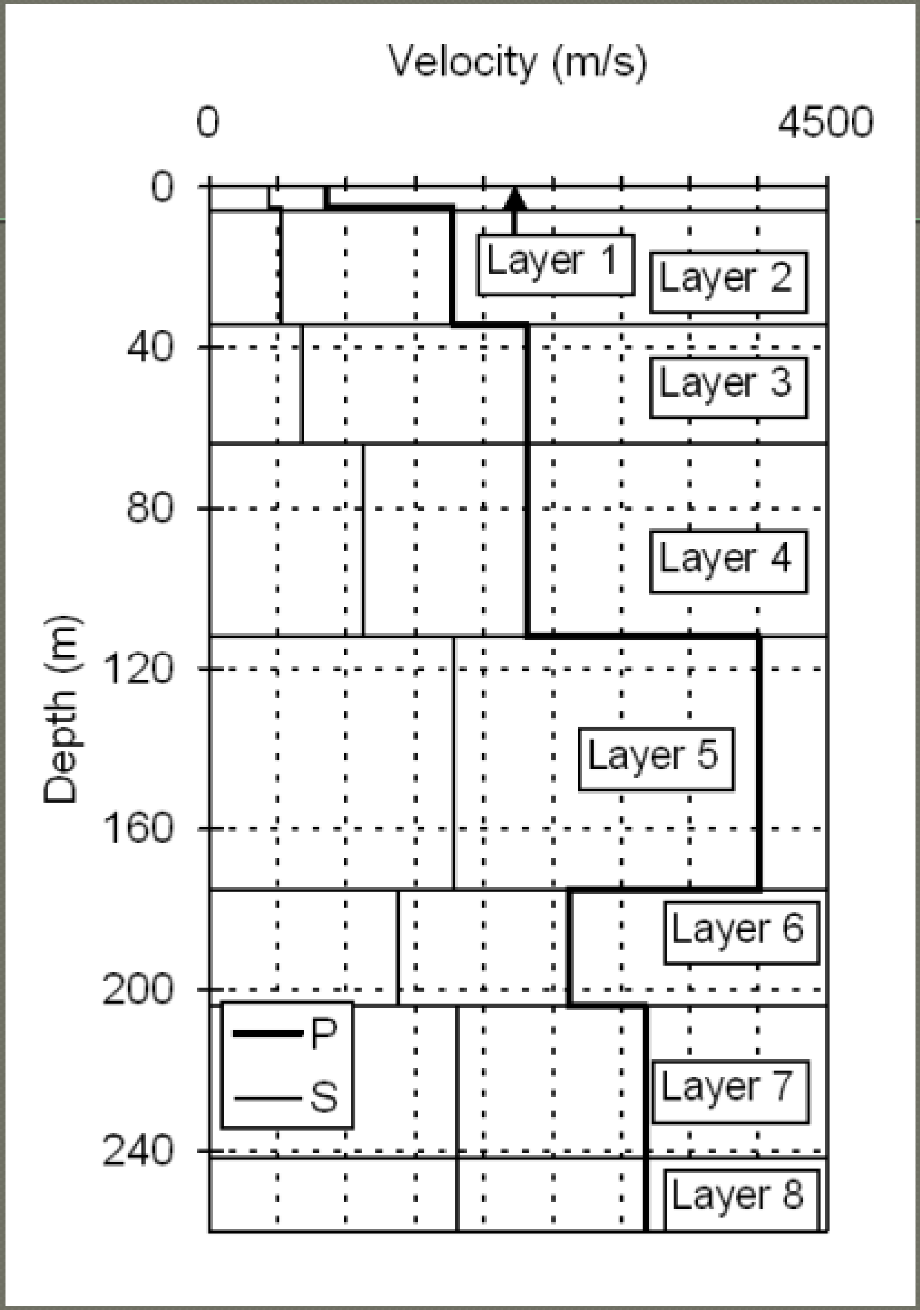
0.06
0
-0.06
400
0
-400
ε_v (%)
Acceleration (m/s²)
Volmetric strain
Surface UD Acc.
Input UD Acc.
Input Amp. 19.6 m/s²
(a)
150
0
-150
50
0
-50
σ_m (kPa)
Acceleration (m/s²)
Ave. conf. stress
Surface UD Acc.
Input UD Acc.
Input Amp. 19.6 m/s²
(b)
5.00E-01
6.00E-01
7.00E-01
8.00E-01
Time (s)
80
0
-80
-160
-0.16
-0.08
0
0.08
ε_v (%)
Input Amp. 19.6 m/s²
Initial slope
K=182 MPa
Nonlinear Elastic response
(c)

Nonlinear Volumetric Mechanism: Evidence from the Field

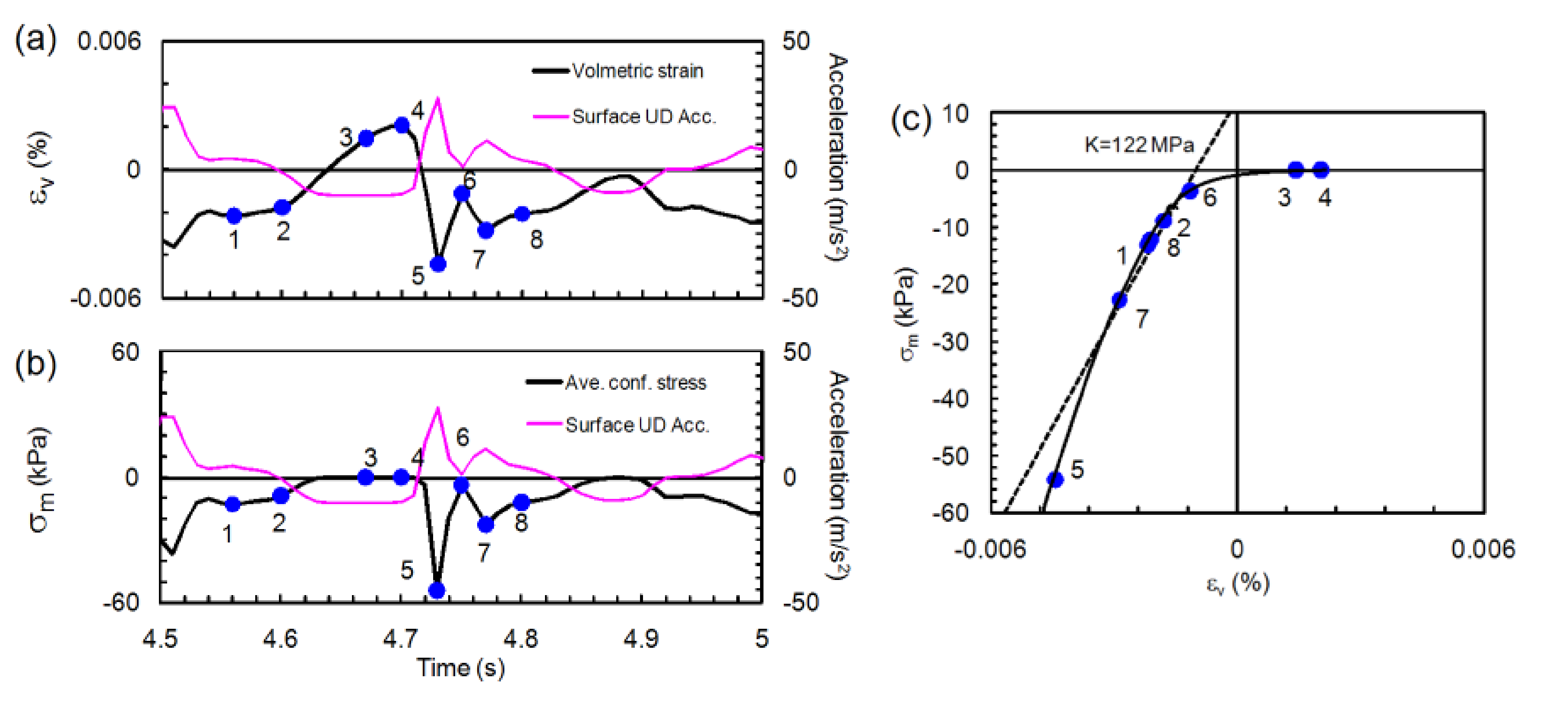




Surface
UD
Max. 61.2 m/s²
Min. -11.2 m/s²
Computed
Surface
UD
Max. 38.7 m/s²
Min. -17.0 m/s²
Measured
Input
UD (INPUT)
Max. 6.38 m/s²
Min. -6.82 m/s²
Measured
Acceleration (cm/s²)
3000
0
-3000
0
5
10
15
Time (s)

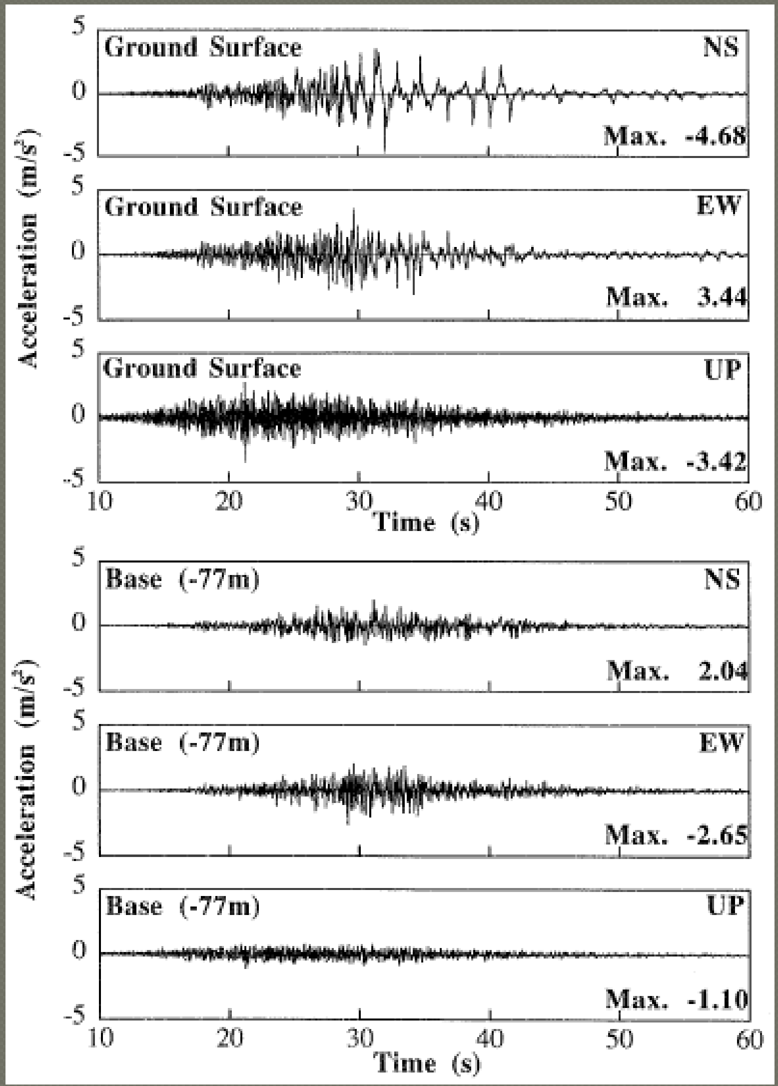


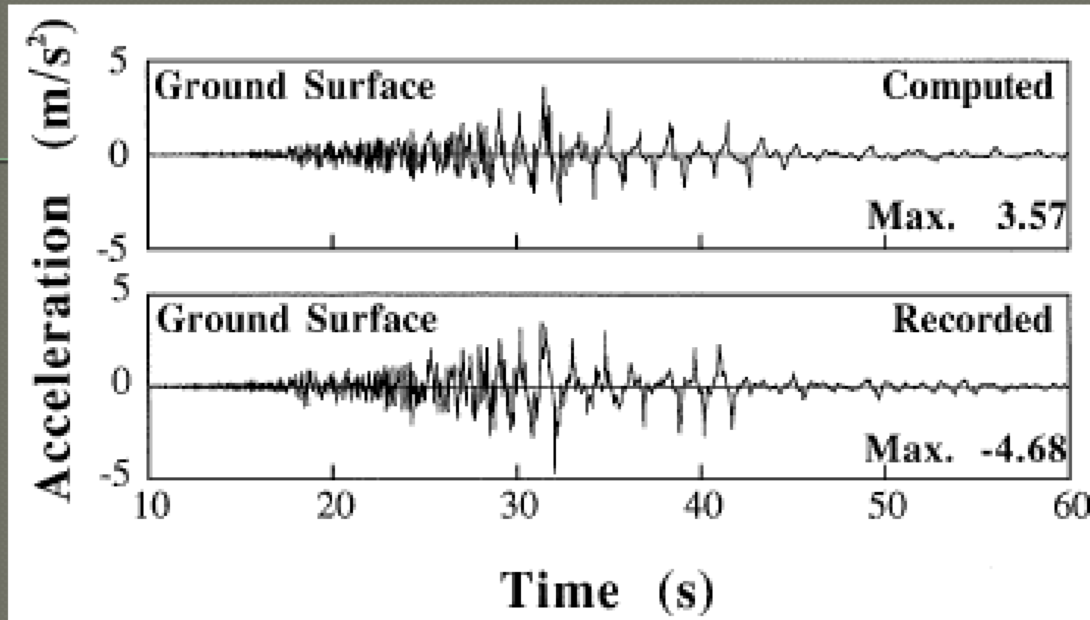
Velocity (m/s)
0
4500
0
40
80
120
160
200
240
Depth (m)
Layer 1
Layer 2
Layer 3
Layer 4
Layer 5
Layer 6
Layer 7
Layer 8
P
S

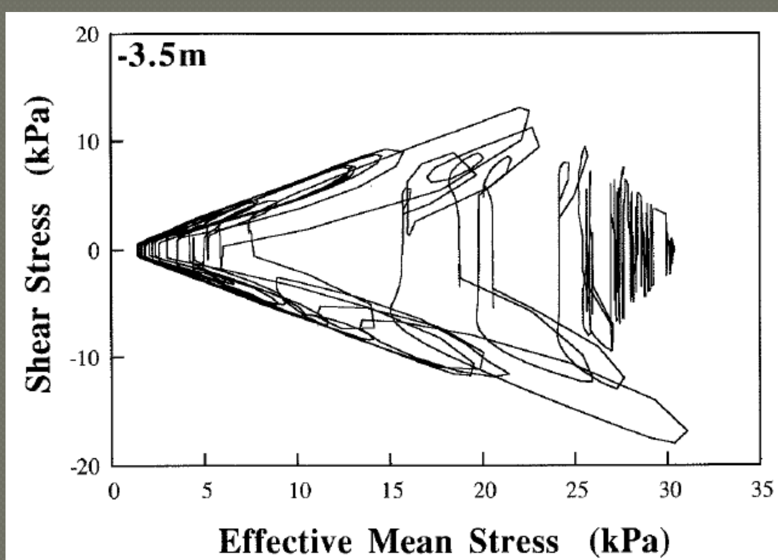


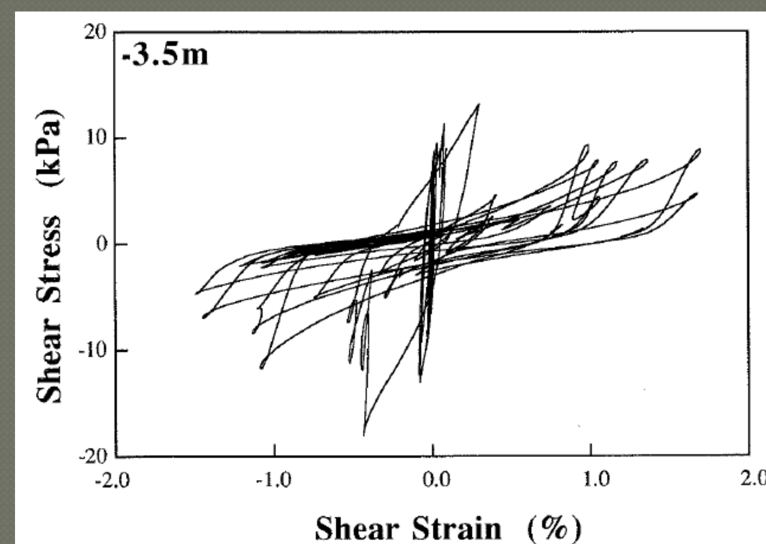
(a)
Volmetric strain
Surface UD Acc.
εv (%)
Acceleration (m/s²)
(b)
Ave. conf. stress
Surface UD Acc.
σm (kPa)
Acceleration (m/s²)
Time (s)
(c)
K=122 MPa
σm (kPa)
εv (%)

Mechanism of Spikes in Site Response

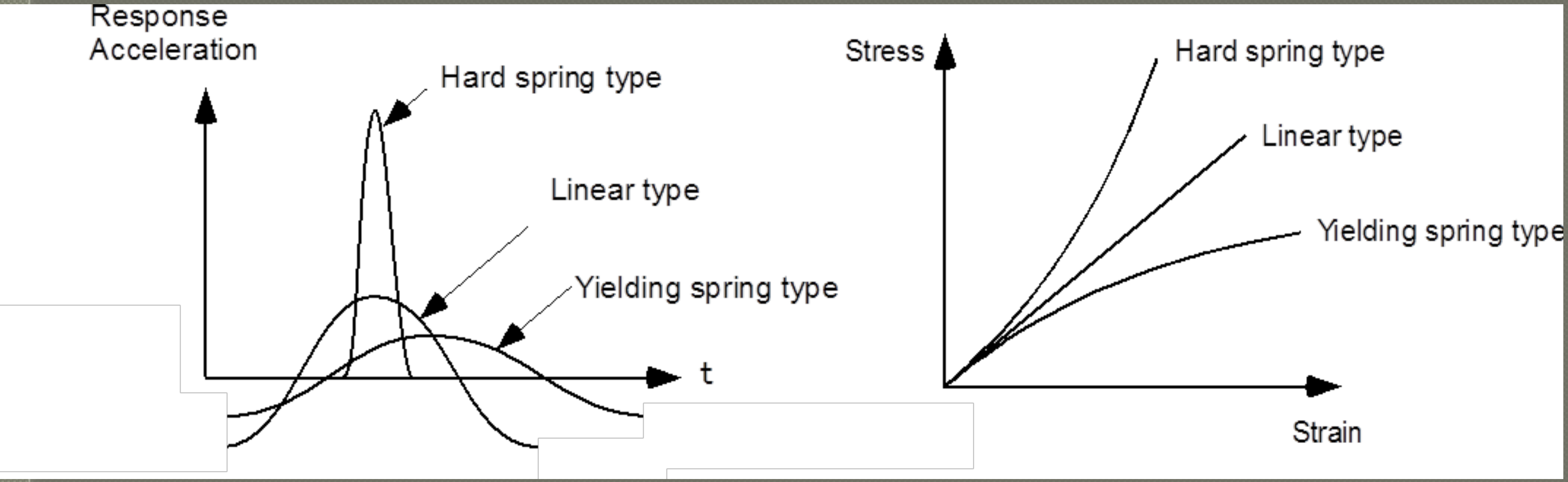



Acceleration (m/s²)
Ground Surface
Computed
Max. 3.57
Ground Surface
Recorded
Max. -4.68
Time (s)


-3.5m
Shear Stress (kPa)
Effective Mean Stress (kPa)


-3.5m
Shear Stress (kPa)
Shear Strain (%)

Nonlinearity



Conclusions

- Gravity→confining stress in the soil
- Strong vertical motion (large volumetric strain)→nonlinear volumetric response
- A peak acceleration of 4.0g (2008 Iwate-Miyagi Inland, Japan, earthquake)
- Nonlinearity of hard spring type→spikes in acceleration